Spring 2021

Mathematical Realism From Reflectance Physicalism

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by

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Submitted in Partial Fulfillment of the Requirements

For the Degree of Doctor of Philosophy in

Philosophy

College of Arts and Sciences

University of South Carolina

2021

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DEDICATION

To my parents

ACKNOWLEDGEMENTS

I thank Michael Stoeltzner for being the best mentor of my professional life, a tireless reader of every abstract, draft, revision, and cover letter that I could send him, and a recommender and facilitator of so many opportunities. I also thank the University of South Carolina Philosophy Department faculty as a whole, for their great patience in letting an engineer come up to speed with philosophical discourse, the framing of theses, and the selection of paper topics. My committee members George Khushf, Chris Frey, and Anjan Chakravartty each spent a lot of time with my work and pointed out pitfalls I had not considered, or just as well, explained which parts were clear; I am grateful for their insights. As I promised verbally to provide, thanks are due also to the owner and staff of the marvelous Drip Coffee shop in Five Points, where for years I wrestled with the ideas expressed in these pages. My morale also would have suffered without the Palmetto Swing Dance Association. Lastly, I thank my family for their enthusiastic support: my father for encouraging me to take a risk and pursue my dream, my mother for her steadfast attention to my education through the years, my siblings and siblings-in-law for the game nights during my much-needed summer and winter breaks, and my nephews for the endless music, smiles, and trips to the park.

ABSTRACT

Do mirrors remain "reflective" objects in the dark, or does light shining onto a mirror instead give the mirror its reflective ability in the moment? More than an idle barstool question (like whether a tree falling in an abandoned forest makes a sound) the intrinsicality or light-independence of reflectance carries considerable philosophical import, since some philosophers reduce the human-visible colors to intrinsic surface reflectance. My dissertation, while remaining neutral on the best definition of color, argues that the received view of reflectance leaves it conceptually regressive and thus non-ascribable to surfaces. Rendering reflectance intrinsic to surfaces, I argue, requires a mathematized redefinition of reflectance, the literal interpretation of which implies a limited mathematical realism, itself a millennia-old philosophical bugbear. Without advocating mathematical realism *per se*, my thesis implicates a variety of current debates in scientific structural realism, metaphysical dispositional realism, mathematical nominalism, mathematical explanation, and even aesthetics, thanks to the philosophical precedent of reducing color to reflectance.

Here is the argument whose implications I explore throughout my dissertation chapters. The received definition of reflectance is the per-wavelength efficiency of a surface to reflect "pulses" of light, pulses being finite-duration propagations. I object that according to a well-documented law of nature, all electromagnetic pulses exhibit an inverse relationship between their duration and bandwidth, and that this relationship generates a vicious regress of the purported reflectance value at any wavelength. I block

the regress by redefining “pulses” as superpositions of Fourier harmonics, which are infinite-duration monochromes. If harmonics reflect from surfaces, however, then they must be real.

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INTRODUCTION

1. Personal Roots of the Mathematical Ontology Question

There was an episode of *Sesame Street* that impressed me when I was young, the end of which featured the announcement, "This episode was brought to you by the number three"; concurrently with this announcement, a puppet hoisted either a placard with a "3" on it, or a stuffed felt numeral "3," I cannot quite remember.[1] But I do remember realizing that the sponsorship was a joke, and an irreverent one. *The* Number Three would not deign to appear on this noisy show, I mused, because only jokers appeared on this show, and because even *adults* believed in the Number Three. The Number Three was real. I was a Platonist, and that was that.

In hindsight, I think that my platonism was linguistic. I learned about negative numbers at home, before they were introduced in my schooling, with the result that when my teacher wrote "3 − 5 = ______" on the board, and I answered, "−2," the teacher replied, "No, we can't do that one."[2] *Yes we can*, I thought, because −2 is as real as 3; writing sums is not the same as writing short stories about what happened on Halloween;

[1] Segal (2019, 1) says the puppet was Ernie, but I wrote this reflection before reading Segal.

[2] The teacher then backpedaled and explained that we *wouldn't* do that one, but "can't" was the teacher's first response.

every possible solution is an actual solution in mathematics.[3] Numbers and arithmetic are *about* something more serious than fiction, I thought, since they keep track of money, fuel in the car, siblings, and servings of food. In the ancient world, more sophisticated adults than I harbored similar intuitions (if not necessarily linguistic ones). The Pythagoreans gave the numbers 1 through 10 cosmological significance, attributing to them the principles of line, shape, and existence itself (Raven 1948, 140). Other smart people to this day affirm a human-independent reality of mathematics, either because of the (apparently) necessary truth of mathematical claims, or because of the indispensability of mathematical terminology to our best science. As Russell Marcus (2015) puts the point, not only are mathematical claims among our most "secur[e]," but they appear in scientists' most rigorous attempts to "tell us what there is" (90); so why deny an ontological dimension to mathematics?

Mathematical nominalists, on the other hand, possess ready answers to this question. Some nominalists object that although both mathematical terms and unobservable terms like "electron" appear in empirically confirmed theories, one can deny on scientific-methodological grounds that mathematical referents are posited, measured, confirmed or disconfirmed to exist (e.g., Sober 1993). Others argue that applied mathematics reduce in every instance to descriptive language about the non-mathematical physical world (Saatsi 2011), the usefulness of that language never quite amounting to existential import. One way to deny the existential import of mathematical descriptions is to translate mathematical terms into relations of physicalist predicates

[3] I mean that "−2" and "−1 + −1" are both possible and therefore actual solutions to "3 – 5 = ______." By contrast, not every possible monster in a story shows up in the final scene.

(Field 1980); another is to deflate mathematical claims into rules or procedures that are neither true nor false, such that assigning real referents to them is a category mistake (Azzouni 1994: 186).

My dissertation, while taking no *general* position on mathematical realism or nominalism,[4] identifies a widely overlooked route to mathematical realism: the conditionalization of mathematical realism on *property* realism. Granted, set theorists know well a trivial version of this conditionality. Simply nominalize a given physical property into the set of individuals who possess that property, and you become committed to the existence of a set, which is a mathematical object (Marcus 2015, 55). My approach is different. I develop an indispensability argument for mathematical realism, without relying on either the Quinean version that reifies the referents of bound variables in true first-order theories, or on the Enhanced Indispensability Argument (EIA, from Baker 2009) that infers the existence of the referents of the mathematics indispensable to our best scientific explanations. Rather than theorize as a scientist or explain anything, I simply attempt in good faith to be a property realist of a particular stripe—a reflectance realist, reflectance being the property of a surface to reflect light—and from this attempt I discover the need to posit real mathematical entities (Fourier harmonics, or infinite-duration sinusoids) and mathematical properties (the infinite duration of harmonics) to retain reflectance as a conceptually coherent and non-regressive property at all.[5]

[4] My realist thesis is conditional rather than general. Hence, I reject the agnosticism of Balaguer (1998), who argues that "there is no fact of the matter" about mathematical realism and the truth of mathematical language (178).

[5] Reflectance pertains to the entire electromagnetic spectrum including radio, radar, and X-rays, which propagate transversely. Further research is required to determine if my argument applies also to longitudinal sound waves. See chapter 2, §2, for an ontology-free introduction to Fourier analysis.

By "conceptual coherence," I mean the very weak condition that ascriptions of real properties should not be self-contradictory; reflectance should turn out neither to mean nor to be expressed as something analogous to a "non-cubical cube," but I will argue that such contradictions indeed precipitate from the philosophical received view of reflectance advanced by David R. Hilbert (1987; Byrne and Hilbert 2003). Because I have no *sophisticated* argument for forbidding grammatical or semantic contradictions in real property ascriptions, I briefly defend that stipulation now. My guiding observation is that ascribers of real properties do not reason, talk, or act like ascribed properties can change their meaning in the course of a sentence or analysis about them. When Frank Jackson and Philip Pettit (1990) ascribe "fragility" to a vase, for example, they mean that it shatters like-so upon impact, not that "fragility" means or could arbitrarily mean "does and does-not shatter" (barring metaphysical tricks like nested dispositions, finks,[6] Goodmanian bent predicates, etc.). An unobjectionable extrapolation from this observation is that analytic philosophers generally avoid double-talk about ascribed properties in a single analysis. Of course, quantum dualities and benign circularities[7] abound, and they violate conceptual coherence as I minimally define it, but much of the *property realist* talk within the philosophy of science exhibits unwavering consistency. Of course, I do not wish to over-emphasize the role of language for my argument. My argument engages explicitly with linguistic practice in chapter 4 on Jody Azzouni's

[6] See Martin (1994, 2-3). A finked wire has the disposition to shunt its electric current to an auxiliary conductor, but thanks to a finking device, the wire possesses this disposition *only when* the auxiliary conductor is actually touching the wire. Thus, one could say that this disposition is-and-is-not the wire's state of conducting current, or of "being live"; but I elaborate this argument no further.

[7] Marcus (2015, Chap. 11) endorses a benign circularity about epistemic justification.

nominalism, and in chapter 7 on the conservativeness of mathematics defended by Hartry Field, but applies nevertheless outside of those domains. Nor have I any formal account of what a "semantic contradiction" is, for example in terms of propositions. I merely argue that realist property ascription should not yield the *prima facie* contradictions that I have flagged for reflectance.

Here someone might object that my prejudice against apparent contradictions is unwarranted. Considering especially that reflectance is understood by many to be an "intrinsic" disposition,[8] one might contend that a "logically uninstantiable" property like "being a non-cubical cube" nevertheless counts as an intrinsic property of an object (Marshall 2020, §1). One definition of intrinsicality suggesting this result is the following:

> The property of being F is intrinsic iff, for any x, had x been F, then it would have been that x being F was a matter of how x and its parts were and how they were related to each other, as opposed to how they were related to other things and how other things were. (Marshall 2020, §3, numbering removed)

Hence being a non-cubical cube, whatever that means, is intrinsic to entity *x*, if being cubical and non-cubical *is* all about the parts of *x* and their internal relations. In the first-order kind of reply appropriate here, I can counter-object that no one analyzes reflectance as if it resembled a "non-cubical cube," and that if reflectance does bear such a resemblance, then the philosophical community is decades overdue in coming clean about it. Secondly, one might ask the objector if analogizing reflectance to a non-cubical

[8] Hilbert (1987, Chap. 4), Ellis (1992, §4), Jackson (1998, 87), Byrne and Hilbert (2003), Chakravartty (2007, Chap. 2), Azzouni (2010, 30), and Isaac (2018, 521, 524).

cube is any *less strange* than the mathematical realism that follows from my *resolving* that contradiction.

In sum (and in preview), I argue in all but the final chapter of this dissertation that the received philosophical view of reflectance is conceptually incoherent and thus non-ascribable as a real property.[9] Restoring coherence requires a mathematical redefinition of reflectance, I argue, and the mathematical constituents (Fourier harmonics) of this redefinition cannot be nominalized away, on pain of conceptual regress (resulting in incoherence). Thus, my limited indispensability thesis: positing real reflectance entails a limited class of real mathematical entities (Fourier harmonics). I now embark on the task of defusing two relevant objections to this thesis: (i) that my approach reduces straightforwardly to Quinean or explanatory indispensability (EIA), and (ii) that some account of idealization likely nullifies my mathematical *realism* conclusion. Defeating these objections is not critical to my project, since my chapters reveal important implications of the reflectance regress even if (i) and (ii) are true. Nevertheless, my thesis gains momentum if I can undermine (i) and (ii) than if I cannot.

2. Background on Mathematical Indispensability

Mathematics is trivially indispensable to theoretical systems that axiomatize mathematical existence claims. Peano arithmetic, for example, treats as a first principle the claim that for all existent natural numbers X and Y, X plus the successor of Y equals the successor of X-plus-Y; hence within Peano arithmetic, "successors" exist, and precisely *as* natural numbers. The mathematical nominalist may side with Rudolf Carnap

[9] Chapter 2 presents this argument more comprehensively, with more detail and defense than in any other chapter. Those wishing to avoid repetition may skip the clearly marked sections of chapters 3-8 in which I re-present the argument.

(1950), however, and deny that mathematical existence claims possess any *meaning* outside the "framework" in which mathematical assertions are best verified—the strictly analytic framework that disregards empirical fact (24). Hence the question, "Do numbers exist?" has for the Carnapian no meaning in the commonplace context in which might be asked, "Do any coffee tables from the 1700's exist?" And yet, just as the Carnapian deploys framework relativism against the mathematical realist, so the Quinean redeploys framework relativism against the Carnapian: if any linguistic framework goes, the Quinean argues, then the empirical science framework is no *worse* an arbiter of mathematical ontology than is the Peano system, out of the gate. That mathematical referents enjoy confirmation alongside physical referents within our best scientific theories is one tenet of the influential Quinean indispensability (QI) argument.

W. V. O. Quine never explicates QI completely in a single text (says Marcus 2015, 41), but well-known is one motivator for Quine's bold view: his rejection of the analytic-synthetic distinction between sentences, the distinction to which Carnapians appeal in answering mathematical existence questions. Quine (1951) problematizes the Carnapian appeal to a sheerly analytic framework for answering mathematical existence questions, in part by arguing how unclear the concept of analyticity is. Traditionally construed, analytic sentences are those whose subjects and predicates are inter-definable or equivalent, like "All bachelors are unmarried," whereas synthetic sentences appeal to experience: "It is raining." Quine wonders, however, what is different about these sentences as kinds. Calling analytic those sentences which would be self-contradictory to deny does not clarify matters, Quine argues, but only negatively reconstrues analyticity as non-self-contradictoriness (20). Avoiding self-contradiction does not explain how an analytic sentence specifically attributes to its subject "no more than" the subject's concept metaphorically "contain[s] . . ." (20-21).

More broadly, Quine (1951) agrees that sentences are true only if they get the language right and the facts right. "'Brutus killed Caesar'" is false if "'killed'" means "'begat'," but also if Caesar never lived in Rome, and Brutus assassinated only Romans (34). Could analytic sentences be those true entirely in virtue of their grammatical meaning, in abstraction from the facts (21)? Quine inclines to a negative answer by arguing that "meaning" amounts to synonymy, and that synonymy is no easier to define than analyticity (23).

The indispensability argument (QI) gains traction, then, when Quine (1951) argues that the dubious analytic-synthetic distinction has been smuggled into the "verification theory of meaning" (35) by which Carnapians answer existence questions. For according to the verification theory of meaning, "the meaning of a statement is the method of empirically confirming or infirming it" (35); but if the method of confirming analytic statements just is composing or uttering them (35), and yet we lack an *account of* the analytic-synthetic distinction, then the way is open to suppose that logical truths and the principle of non-contradiction are *not* analytic, or are amenable to revision contingent on experience. The way is additionally open to suppose that the mathematical terms of a scientific sentence are confirmed when the sentence as a whole is empirically confirmed.

Indeed, Quine argues positively that mathematical knowledge *is* justified by empirical science. For discussion, I follow Marcus's (2015, 42) parsing of Quine's argument (QI):

> QI1: We should believe only the theory which best accounts for our sense experience.

> QI2: If we believe a theory, we must believe in all of its ontological commitments.
>
> QI3: The ontological commitments of any theory are objects over which that theory first-order quantifies.
>
> QI4: The theory which best accounts for our sense experience quantifies over mathematical objects.
>
> QIC: We should believe that mathematical objects exist.

Two primary assumptions support premises QI1-QI4. The first is naturalism, assumed by QI1 and QI4 to determine existence according to physical sensation. The second is the confirmational holism engendered or encouraged by Quine's rejection of the analytic-synthetic distinction. QI2 is holist for endorsing *all* the ontology of a theory, and QI3 stipulates a specific method for determining the ontology of that wholly confirmed theory about our sensations.[10]

Virtually all premises of QI have proven controversial in past decades, and it falls beside my purposes to recount that history here. I instead review in the next section some main objections to QI, to reveal why QI has been largely abandoned for the Enhanced Indispensability Argument (EIA). Generally speaking, and as I will elaborate in later sections, my mathematical indispensability argument from property ascription differs from QI because I am not trying to interpret *theory*, as much as I am attempting to render a physical property's definition coherent, so as to ascribe it as real. Yes, I employ the scientific theory of Maxwellian electromagnetics in my definition of reflectance, but I do not look to it for my ontological commitments, before I look to the language of a

[10] Some authors (Ivanova 2015, 86, 88) associate Quine's holism with Pierre Duhem's, but to elaborate that connection would be a digression from this already lengthy introduction.

coherent property ascription (i.e., the property's definition) for *its* ontological commitments. I identify a conceptual regress for reflectance ascription, which is unlikely to arise within classical electromagnetic theory proper (since property ascription is not a scientific discipline—there are no International System (SI) units for ascription, dispositionality, categoricity, intrinsicality, tropes, universals, etc.). Thus, while my activities in this work do not include confirming theories of experience, my argument proceeds in the spirit of QI3; I do not employ first-order logic, but I think that double-talk about property ascription is out-of-bounds (see section 1), and one of Quine's purposes for QI3 is to avoid double-talk about ontology, as I will explain in due course.

3. Controversy with QI, and the Migration to EIA

Problematizing naturalism (QI1) will not render my thesis any more plausible than it otherwise is, and not many mainstream authors attack QI1, so in this section I leave QI1 aside and review contemporary objections to QI2-QI4.

3.1 Rejecting QI2 Holism

3.1.1 Elliot Sober and Contrastive Empiricism

Confirmational holism (QI2) has been resisted on a number of grounds. Elliot Sober (1993) contends that theories are neither confirmed nor infirmed "absolutely" as stand-alone proposals, but rather "relatively" to competing theories (39). Hence in the development of science, the alleged truth of a theory matters less than if it has been tested, and its alternatives defeated. Sober casts these tests and defeats in terms of likelihoods, an epistemology he calls "contrastive empiricism":

$$P(O|H_1) > P(O|H_2) \text{ if and only if } P(\sim O|H_1) < P(\sim O|H_2) \qquad [1]$$[11]

Observation O (usually a detection performed with instruments) in Equation 1 supports hypothesis H_1 over H_2, iff the failed attempt to detect O would lead scientists to favor H_2 to H_1. Similarly, Sober argues, were mathematical sentences M indispensable to every hypothesis H_1, H_2, . . . H_x under test, then M itself would not be tested, and so it would be neither confirmed nor infirmed; M would be "a background assumption" needed to test the hypotheses (45). He adds that *contra* the tradeoff prevalent in Equation 1, scientists and philosophers do not in practice abandon any applied mathematics M_1 when observations favor a theory using M_2; thus, the mathematics are not really tested, and so are not really confirmed or infirmed (53).

Mark Colyvan (1999) objects that some contrastive empiricist conclusions—such as that two equivalent applied mathematical expressions, or a mathematical expression and its nominalist counterpart, will not yield probabilistic differences between them and so will not be tested—are actually demerits against contrastive empiricism in favor of scientific realism, since the adjudication of competing theories must then proceed by realist criteria like parsimony and "elegance," etc. (325-326). He presents additional arguments to suggest that contrastive empiricism "conflict[s] with actual scientific practice" (326), but my purpose here is not to adjudicate the dispute; I am only summarizing influential anti-Quinean accounts like Sober's.

[11] From Sober (1993, 44).

3.1.2 Joe Morrison on Problematic Assumptions about Confirmation

Another opponent of QI2 holism is Joe Morrison (2012). While side-stepping Sober's and others' empiricist scruples about the *truth* of mathematized theories, Morrison finds QI2 overly broad in its alleged confirmatory power. Specifically, Morrison interprets QI2 as an implausible "*Confirmation Thesis (CT)*" entailed by the conjunction of a more plausible "*Prediction Thesis (PT)*" with a faulty assumption about confirmation itself. PT asserts that "Only whole theories imply observations" (266), and Morrison finds this thesis plausible because it amounts to the "theory-ladenness of observation . . ." (271). As I understand the theory-ladenness of observation, not all parts of an experiment or measurement are themselves varied against a control or measured; without a theory (implicit or explicit) of measurement in place, observation would be no observation but noise or a false or meaningless reading.

The CT alleged to support QI2, on the other hand, says that "Observations confirm entire theories, not individual parts thereof" (269). If true, CT would support the Quinean thesis that not only the physical posits of a theory about our sensations are confirmed under test, but that indispensable mathematical posits are confirmed, too. Morrison (2012) objects that CT only follows from PT when one assumes some version of the now-obsolete hypothetico-deductive (HD) theory of confirmation, the "principle . . . that for α to confirm β it suffices for α to be deductively derivable from β" (271). Not only is this "extra-holistic machinery" of HD unmotivated by indispensabilists, Morrison argues (271), but it suffers classic problems of its own. For example, because "all swans are white entails that all swans are swans" (272), HD unintuitively presents the swan-ness of swans as confirmation of their whiteness.

Even in the event that HD could be rendered non-controversial, however, Morrison sides with Sober (1993) in denying that QI2 necessarily confirms mathematical entities on a par with other unobservables like electrons. Morrison (2012) instead finds CT "compatible" with the supposition that observations provide "differential support" to particular hypotheses of a confirmed theory (270). As Sober employs likelihoods to differentiate support for hypotheses, so Morrison holds that likelihoods and Bayesian probabilities can be shown to support only the non-mathematical hypotheses of whole theories confirmed by observation.

3.1.3 Kenneth Boyce and the Non-Confirmability of Conservative Mathematics

Even more recently, Kenneth Boyce (2020) denies the confirmability of mathematical statements, but he does so without joining Sober's (1993) ranks,[12] and without raising any qualms about hypothetico-deductivism. Boyce argues instead that if (1) the mathematics applicable to science is conservative (defined in the next paragraph), and (2) "the empirical relevance of mathematical statements . . . is mediated by their non-mathematical consequences" (12-13), and (3) statements are confirmed by generating empirical expectations (relative to background information) that come to pass, then mathematical statements applied to science are never confirmed (contra QI2). Because I discuss parts of this argument in chapter 7, it pays to expand some of Boyce's claims here.

[12] Against contrastive empiricism, Boyce (2020, 12) remarks that we cannot test objects that we nevertheless believe to exist outside our light cone, and (citing an argument from Mark Colyvan) that the dispensability of mathematics to science could count as an "empirical" reason to call such mathematics false.

Hartry Field (1980, Chap. 1; 1985) famously argues that applied mathematics need not be true *if* it is conservative, *viz*., if conjoining mathematical statements M_T* to nominalist physical statements N* does not logically entail any *new* nominalist statements not derivable from N* alone (I use Boyce's 2020 notation). Boyce (2020) motivates this thesis (1), which he calls **Conservation**, by arguing that applied mathematics *should* be conservative. Conservativeness accommodates the realist intuition, for example, that mathematics is necessarily true *a priori*.[13] Boyce further enjoins the philosopher of science not to *pre-judge* physical possibilities *via* the conservative mathematical language that merely facilitates empiricist inquiry (14). While I think that this precaution begs the question against the Platonist who believes that the world is mathematical and that scientists observe it to be so,[14] I acknowledge that showing some piece of applied mathematics to be non-conservative is no easy affair (see my attempt for Fourier analysis in chapter 7).

Boyce's second premise (2) he calls

> **Mediated Relevance**. For any background mathematical theory suitable for use in science, M_T*, any mathematical statement, M*, that is either implied by M_T* or by M_T* in conjunction with various nonmathematical statements, and any body of non-mathematical background information and auxiliary assumptions, B*, if M* generates empirical expectations relative to B* that are not generated by B* alone, then there is some non-mathematical statement that is logically implied by M*&M_T*&B* that is not logically implied by B*. (Boyce 2020: 14, formatting mine)

[13] Although as Melia (2006, 203-204) points out, those who rely primarily on QI to be mathematical realists are unlikely to place much stock in the Platonist intuition that mathematics is *a priori* true; for QI is an *a posteriori* justification.

[14] Views along these lines include Berenstain (2017), Marcus (2015, Chapters 8-11), Lyon (2012), Tegmark (2008), Barrow (2004), and Chakravartty (1998), among many others into antiquity.

A central insight of **Mediated Relevance** is that applied mathematics can in many cases generate empirical expectations and partially entail novel nominalist statements *without* violating **Conservation**, since deriving new nominalist statement B^0 from M^* & M_T^* & B^* is not the same as deriving it from M_T^* & B^*, only the latter of which derivation violates **Conservation**.[15] When M^* is "mixed mathematical" (14) and so derives not from M_T^* alone but from M_T^* conjoined with another non-mathematical statement (not in B^*), then empirical expectations and nominalist statement B^0 may be generated while satisfying **Conservation**.

An example of conservative applied mathematics partially entailing new nominalist sentences is Alan Baker's (2005; 2009) much-discussed explanation[16] of the North American *Magicicada*'*s* 13- or 17-year dormition cycles (duration being species-dependent). *Ex hypothesi*, one species of *Magicicada* emerges from the ground to mate only every 17 years, because (a) ecological conditions allow the cicadas a dormition window of 14-18 years (Baker 2009, 614), and (b) minimizing lifecycle intersections with those of predators and non-ideal mates within the window of (a) is evolutionarily advantageous. Since (c) the prime-period lifecycle of 17 years minimizes the intersection frequency mentioned in (b), then the prime integer 17 seems to possess **Mediated Relevance** to the explanandum, in what Boyce (2020) calls one of two senses: "vacuously" or "non-trivially" (16). The former obtains when the number theory of (c) turns out to be nominalizable and dispensable to the explanation; otherwise, the

[15] Thanks to Kenneth Boyce for clarifying this point in email correspondence.
[16] The reader may object that I am jumping ahead to explanation when QI is about theory proper. Because Boyce (2020) interprets some explanations to involve empirical confirmations, the present example in the main text seems appropriate. I revisit Baker's example in an explanatory context in section 4, and in chapter 5.

Mediated Relevance is non-trivial, and the mathematical sentence at stake (I presume: "a 17-year period minimizes disadvantageous intersections") could in principle be confirmed (16).

Without taking a clear position, by my reading, on whether the cicada example employs **Mediated Relevance** vacuously or non-trivially, Boyce emphasizes that in many instances, mathematical sentences logically facilitate new nominalist conclusions from background nominalist theory and assumptions, without indicating anything about mathematical truth and reference (15). The conjunction of ((A→B) & A) yields B, for example, even if (A→B) is never in fact true. Hence by his premises (1) and (2) already discussed, and by his third premise (3) about how sentences incur confirmation (omitted here for brevity), Boyce generates a contradiction from the assumption that mixed-mathematical statement M* (generally construed—not specifically the cicada version) is confirmed by the coming to pass of the empirical expectations that it generates (§5). The upshot is that in proportion to the plausibility of **Mediated Relevance** and Boyce's premise (3) about empirical expectation, the proponent of QI appears pressed to undermine **Conservation** rather than contrastive empiricism, and disputes over hypothetico-deductivism become moot, since *however* confirmation works, **Conservation** inoculates mathematics from confirmation and disconfirmation. Because my arguments do not depend on confirmation theory, this section closes my discussion of QI2.

3.2 Rejecting QI3 and First-Order Regimentation

3.2.1 Joseph Melia and the Scientific Weasel

Another common objection to QI is that QI3 ("The ontological commitments of any theory are objects over which that theory first-order quantifies") is too strong. Joseph Melia (2000) contends, for example, that theorists can and do "weasel" away from ontological commitments in their professional discourse; they "assert that P whilst denying a logical consequence of P . . ." (466). Logical consequences of P might include mathematical consequences of an ontological sort (e.g., "there are functions," "there are sets"), but Melia insists that it is commonplace for theorists to tacitly take these claims back, after asserting P.

Melia (2000) supports the weaseling thesis by arguing that some conservative, mathematized versions T* of nominalist physical theories T entail *nominalist* ontological consequences that are *not* nominalistically expressible. The nominalist consequences are instead only mathematically expressible, and so those mathematical assertions can be rightly weaseled or rescinded after the fact. Specifically, Melia analyzes a theory T* about indivisible atoms that combine mereologically as parts of larger regions of atoms. By premises too complex to reiterate here, theory T* entails the existence of physical regions R that are "infinite and coinfinite," meaning that R and its complement each contain an infinite number of atoms (460). T* entails these regions by conjoining mathematical structure to T, a mereological theory whose regions and their complements each contain only finitely many atoms. While R exists as a non-mathematical reality according to T*,[17] the existence of R cannot be expressed nominalistically; for *asserting* that R exists requires asserting that the atoms of R "are mapped onto the even numbers by some 1-1 function from atoms to ordinals" (462). Thus, Melia holds that the T-theorist can believe in good conscience in the existence of R, while kicking away the mathematical "scaffolding" (469) required to assert the existence of R.

Less sophisticated examples of weaseling include drawing spatial conclusions from a 2-D map while nominalizing the relations of points on that map to points on a 3-D globe that unquestionably obtain (Melia 2000, 468). Melia (2000) denies that we are committed to saying anything about globes when we discuss maps, and he finds it more "charitable" to assume that weaseling is routine scientific practice, than that it skirts QI3 through duplicitous double-talk (469). While the weaseling strategy (i.e., "easy road" nominalism[18]) continues to garner both supporters[19] and detractors,[20] more profitable to consider in this overview is how *essential* for indispensability arguments the anti-weaseling tenet QI3 appears to Marcus (2015), who does not even endorse QI. For Marcus's insights resemble my own, since as I remarked in section 1, my (informal) indispensability argument from coherent property ascription forbids as a matter of principle double-talk about some properties mathematically defined.

[17] Some theorists like Azzouni (1994, 3; 2009, 161) might object that all infinitudes are mathematical, but Melia (2000, 460) and Field (1980, 95, 101-102) allow nominalist construals of infinitudes. I revisit this discrepancy in chapters 4 and 7.

[18] A term coined, I believe, by Colyvan (2010).

[19] Liggins (2012), Azzouni (2012), Yablo (2012), Knowles and Liggins (2015), and Plebani (2017).

[20] Colyvan (2010), Shech (2019), and my chapter 4 of this dissertation.

3.2.2 On the Importance of QI3 for Indispensability Arguments

Simply stated, Marcus (2015) interprets Carnap to be "The Original Weasel" (78) who takes back ontological commitments through substituted linguistic frameworks. While QI3 does not by itself stop the Carnapian from performing these substitutions, Marcus finds QI3 to temper the naturalist thesis of QI1, that "there is no perspective external to scientific theory from which to choose among various formulations of a theory" (82). The utility of QI3 for the Quinean, in other words, is that it nails down ontological commitments *however* scientists happen to endorse or reject theories according to their virtues of simplicity, unificatory power, predictive success, parsimony, and elegance (87). Without QI3, for example, Melia might argue that weaseling away the 1-1 mapping function over even numbers in T* is *more* elegant than not weaseling it (section 3.2.1), thereby leveraging QI1 against the Quinean mathematical ontology. While a number of indispensabilists[21] and nominalists[22] agree that mathematical terminology often renders scientific theories more virtuous than their nominalist counterparts, omitting QI3 allows the Carnapian an apparently fundamental veto power against unwanted ontological commitments.

Although (as mentioned) my argument does not depend on first-order logical regimentation, the present discussion benefits from an examination of Quine's reasons for requiring QI3, since I sympathize with the spirit of that requirement. As Marcus (2015) helpfully summarizes, Quine requires first-order logical regimentation, in part, because first-order logic *itself* possesses theoretical virtues like "extensionality, efficiency,

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[21] Colyvan (2001, 76-86; 2010, §2).
[22] Azzouni (2004a) and Liston (2004; 1993).

elegance, convenience, simplicity, and beauty . . ." (52). More technical advantages of first-order logic include its completeness, and the availability of a model for every consistent theory (53). Quine (2013) denies, furthermore, that "there is" in the English metalanguage possesses *different* meanings for the existence of physical versus abstract objects, and so he finds unlikely any systematic misapplication of the first-order existential quantifier between those objects (222-223). My sympathies lie with this last point; although I will not argue a distinction between "there is reflectance," "there is a mirror," and "there are Fourier harmonics," I will deny that statements analogous to "there is a non-cubical cube" are fair or reasonable in discussions of reflectance (chapter 4).

In the meantime, some might suspect that the Quinean and the Weasel are sometimes up to the same tricks. As Alyssa Ney (2014) explains, first-order logic conveniently accommodates "semantic ascent" (38), or the non-referential linguistic practice of mentioning useful nouns. By mentioning without referring, semantic ascent may look like weaseling, but I understand weaseling to be a backup tactic to semantic ascent, the latter occurring when nominalistic paraphrases are available and ontologically important (if inconvenient) to express, with weaseling transpiring for examples like Melia's region R or 2D-3D spatial relations between globes and maps (section 3.2.1), where paraphrase is impossible or unimportant.[23]

If one discusses animal "species," for example, but declines to believe that "species" exist over and above the members *of* those species, then one "ascends" to

[23] I claim no absoluteness about this distinction, and I recognize that Azzouni (2004a, 182-183) might dispute it.

claims about the members, while using the non-referring noun, "species." Then, when it comes to formulating theorems of what one believes, such as, "'Some zoological species are cross-fertile'," one may first-order regiment the quantification $\exists x\,(Sx \wedge Cx)$,[24] into $\exists x \exists y\,((Lx \wedge Tx) \wedge Mxy) \vee \exists x \exists y\,((Bx \wedge Ey) \wedge Mxy) \vee \exists x \exists y\,((Zx \wedge Cy) \wedge Mxy) \vee \ldots$, where predicates L, T, B, E, Z, and C denote lions, tigers, bears, elephants, zebras, and cobras, respectively; the very long paraphrase about fruitful mating combinations (Mxy) indicates by true disjuncts the cross-fertile species without reifying them.[25] Yes, regimentation can obliterate some mathematical entities from one's ontology (e.g., integers), but indispensabilists are usually aware of how difficult this step is to perform in most applications—including Fourier analysis, as I discuss in section 3.3. Regimentation also notoriously eliminates *properties* like reflectance from one's ontology, and I revisit that insight in section 4.1. The present point is that while I cannot tolerate double-talk about reflectance (section 1), my reason why is not identical to Quine's.

3.2.3 Jody Azzouni on the Ambiguity of First-Order Existential Quantification

For completeness, I would be remiss not to review the objection to QI3 lodged by my chapter 4 interlocutor, Jody Azzouni (2004a, Chap. 3), who denies at length that first-order existential quantification corresponds *very well at all* to "there is" in the metalanguage. I have to escape from his similar "proxy norm" objection in chapter 4, so it pays here to understand some of his motivations against QI3.

[24] Predicate S denotes species, C cross-fertility.
[25] Example lifted from Ney (2014, 43-44).

Firstly, Azzouni (2004a) grants that semantic ascent occurs in "lots of physics" (25),[26] but he denies that "there is" usually merits translation by a quantifier symbol. Alonzo Church (1958) anticipates Azzouni's objection, warning that "correspondence" between metalanguage and the quantifier symbol should not be demanded too strictly, since "ordinary language itself is not accurate beyond a certain point . . ." (1012). But Azzouni (2004a, Chap. 3) considerably magnifies this insight by exploring six interpretational schemes for the "there is" idiom, including paraphrase, Meinongianism, substitutional quantification,[27] metaphor or pretense, "cancellation" (66),[28] and anaphora. By analyzing talk about fictional characters (which according to Azzouni do not exist), he problematizes reference for the first five interpretations of "there is," and eventually settles on "ontologically neutral anaphora" as the least problematic interpretation (78). Hence as Azzouni (2010) elsewhere argues, anaphoric reference sometimes obtains not between terms and truthmakers, but between terms and "truth-value inducers," such as the drawings and artists that render certain claims about Mickey Mouse true or false without referring to any*thing* as Mickey (25). Thus, Azzouni rejects QI3 because "there is" proves quantificationally ambiguous even within science, and this ambiguity gives the Weasel more ontological authority than the Quinean suspects (see Azzouni and Bueno 2016, and my chapter 4).

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[26] Because he thinks that scientific theories are largely false with "*true* implications . . ." (Azzouni 2014, 2999).

[27] Here is Azzouni's (2004a) example: "'Mickey Mouse is a fictional mouse that talks'" (65) entails "'There are fictional mice that talk'" (62), but the "there are" fails to refer because it was inferred logically from a "referentially empty" proper name (66). Mickey Mouse is referentially empty because fictional characters exist "in no sense at all" for Azzouni (57).

[28] Whereby the adjective "fictional" cancels the ontological import of "there is" (Azzouni 2004a, 66).

3.3 Nominalizing Mathematics to Falsify QI4

Lastly, some object to QI by denying QI4 ("The theory which best accounts for our sense experience quantifies over mathematical objects"), the objection being not that first-order quantification is problematic (versus QI3), or that the Quinean mischaracterizes which theories are "best" (versus QI1), but rather that mathematical objects never require existential quantification in principle, because they can always be regimented out of a scientific claim, or nominalized into physical relations.

Well-known and already mentioned are Field's (1980) efforts in this regard, to expunge mathematics from science. Simply put, his nominalization program enables us to deny that we indispensably refer to (e.g.) the temperature of boiling water as "100°C" or "212°F," if we can instead describe the temperature as taking a value **between** two other temperatures, and/or a value **congruent** in its "distance" from a given temperature with the distance between two other temperatures (50). Prescinding still further from this simplified example, Field constructs **betweenness** and **congruence** relations not between temperatures, but between space-time points whose intervening distances map to scalar quantities of interest (like temperature). By a similar strategy, he formulates a nominalization of Newtonian gravitation (Field 1980, Chap. 8), and "suspect[s]" that "all" other "physical[ly] importan[t] . . . . differential equation[s]" could be so nominalized (60).

Legion are the scholarly doubts that Field's program succeeds in various applications beyond (and even within) Newtonian gravity, but I will not recount those objections.[29] I instead need to reckon with the possibility that the Fourier harmonics I find indispensable to reflectance ascription (see section 1, and chapter 2) are among those readily nominalizable by something like Field's program.

Anticipating this inquiry is Michael Liston (1993), who considers the nominalizability of Fourier mathematics into Fieldian predicates (**betweenness**, **congruence**), and who concludes that the difficulty of performing that nominalization is "likely . . . insurmountable" (444). Unlike the "*localized*" relationships between Field's spacetime points, Liston argues, Fourier analysis exploits "global" relationships between the superimposed parts of a complex signal (444). More specifically, a global relationship obtains between the amplitude of a point on a vibrating string, and the amplitudes of that string's vibrational modes or harmonics (437-438). Liston argues that "the closest analog" of these global relationships to Field's localized relationships is the serial coupling of N harmonic oscillators, each oscillator being described by a differential equation whose parameter values "depend on those of the other $N-1$ equations" (444). Considering that up to 99 harmonics may be needed to model a basic square-wave with acceptably small ripple at the peaks, science without Fourier analysis is impractical, and according to Liston, epistemically unreliable (445).

The nominalist might rejoin that Field's program is not defeated by bellyaching, and that apparently taxing derivations (of localized relationships) are no proof of the indispensability of applied mathematics. One advantage of my thesis is that it appears to escape this objection. I argue for the indispensability of mathematics to reflectance

[29] Examples include Malament (1982), Resnik (1985), Liston (1993), and Melia (1998, §§3-4).

ascription *not* because oscillator equations are hard to solve, but because ascriptions of reflectance become *meaningless* and *conceptually regressive* when its definitional mathematics are supplanted (chapter 2). Hence my argument is anti-weaseling before it is anti-Fieldian, but always in the metaphysical property-ascription sense, not in the far more common philosophy of science sense of theory or explanation.

Of course, by mentioning a "sense" of metaphysical property ascription, I do not wish to fall into the Carnapian trap of reifying Fourier harmonics only and uninterestingly in an isolated metaphysical framework ignored by science and philosophy of science. Indeed, I will argue to the contrary that philosophical property ascription in analyses of scientific theory and explanation has been too casual, and that an unexpected and largely unwanted mathematical realism results. Having completed my remarks on indispensability and theory-confirmation, I turn now to indispensability and explanation, a strategy codified in the literature as the Enhanced Indispensability Argument (EIA). Recall that indispensabilists typically migrate to the EIA due to difficulties with QI2-QI4. Lest anyone find my own expository migration too quick, moreover, since scientific theories employ *idealizations* and the Fourier harmonics indispensable to reflectance look like idealizations, I also discuss idealization in the next section.

4. Situating my Argument among the EIA and Accounts of Scientific Idealization

4.1 Background on the EIA

A basic paraphrase of EIA goes like so: If mathematical entities, facts, or properties *M* can be shown to explain a scientifically valuable explanandum, and if no *X* could explain anything if X did not exist, then mathematical entities M exist.[30] Because the explananda valuable to science are typically physical events or physical facts, I use Baker and Colyvan's (2011, 326) terminology to call the EIA a species of "extra-mathematical" explanation. The main motivation for EIA over QI, in my view, is eschewing confirmational holism (QI2), since talk of "confirmation" goes noticeably missing from EIA proposals; but there is also a sense in which weaseling (versus QI3) might be less acceptable in explanation than in formal theorizing,[31] and a sense that some explanations make *intuitively* compelling use of mathematics (versus QI1).[32] Regardless of which motivation is strongest, however, a focus on explanation reduces the mathematical realist's burden of interpreting scientific *theory* with great rigor. As Marcus (2015) puts the point, "nominalism is less plausible if one can show that mathematics plays an explanatory role in addition to its representational role" (126). Boyce (2018, §3) similarly opines that from a scientific realist perspective, explanatoriness trumps representation in picking out the real; EIA transfers this stratagem from science to applied mathematics.

Alan Baker's (2009, 613) formulation of the EIA is as follows:

The Enhanced Indispensability Argument

(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

[30] For readability, I treat mathematical facts, entities, and properties as interchangeable, context permitting.

[31] Consider Colyvan's (2010, §3) anecdote about narrating *Lord of the Rings* but denying near the end that hobbits were really part of the story.

[32] For the record, I find Juha Saatsi's nominalization of Alan Baker's cicada explanation (discussed in my chapter 5) to be deeply unintuitive, but I do not discuss intuition further in this dissertation.

(2) Mathematical objects play an indispensable explanatory role in science.

(3) Hence, we ought rationally to believe in the existence of mathematical objects.

Why believe premise (1)? Baker understands premise (1) to invoke an inference to the best explanation (hereafter IBE; 613). As some argue that only a miracle would render atomic theory predictively successful if unobservable atoms and electrons did not exist,[33] so the EIA theorist infers existent mathematics to be the best explanation for their indispensability to our best scientific explanations. Even if (1) is true, however, premise (2) is far from unobjectionable, and I discuss it later in this section.

To see how EIA works, consider again its *locus classicus*, Baker's (2005) cicadas. Allegedly, (1) we cannot extirpate the "primeness" property of number theory from the true premises of our best evolutionary explanation of cicada behavior (emerging from the ground to breed and die every 13 or 17 years); (2) primeness is mathematical; thus (3) prime numbers, or the primeness property of number theory, literally exist and explain organismic behavior.

Some theorists reject premise (1) of EIA, either because they think that applied mathematical IBE's are invalid (Bangu 2008), or prove vulnerable to counterexamples (Boyce 2018), or because a clear account of the form of "explanation" at stake is wanting (Marcus 2015; Saatsi 2016). Sorin Bangu (2008) contends, for example, that IBEs only work when the explanandum is first assumed to be *true*, such that purporting to explain a mixed-mathematical[34] explanandum with mathematical explanans begs the question.

[33] See Psillos (1999, Chap. 4) for discussion of "no miracles" arguments for realism.
[34] Containing both mathematical and physical (non-mathematical) terms.

Boyce (2018), on the other hand, denies that EIA has left behind all of the confirmation woes facing QI.[35] He interprets the mathematical IBE as assuming "that there is empirical support" from the explanandum event *for* the mathematics indispensable to the explanans (§3), and he presents an IBE counterexample to that assumption (§§4-5; omitted here for brevity).

More neutrally, Juha Saatsi (2016) objects that EIA theorists rarely explicate what kind of "explanation" is operative in EIA premises (1) and (2). He suggests that explanations can be "*ontic*," whereby real properties or entities in the world *are* the explanations for events (1052), or explanations can be "epistemic," in that they "provide understanding" about an explanandum;[36] explanations can also be "modal," Saatsi says, in the sense of being necessary, or more necessary than the laws of our world and its ontic facts (1053).[37] To pause and position my own research, I find it notable that while several accounts of extra-mathematical explanation analyze how laws or properties interact and explain phenomena, relatively few accounts explain *how it is that a given physical property is or can be instantiated at all*. Such is exactly my thesis about reflectance, but theorists typically talk past it, either by endeavoring to explain *events*, or by explaining "properties" like a rainbow's color ordering (Pincock 2011, 20) that are construable as events or phenomena; and even if some of the properties explained are not

[35] As does Marcus (2015, Chap. 7), by my reading.

[36] One version of epistemic explanation, Saatsi (2016, 1049, fn. 6) claims, is deductive-nomological (DN) explanation, an offshoot of the hypothetico-deductivism discussed in section 3.1.2. For a recent DN account of extra-mathematical explanation, see Baron (2019b).

[37] An influential proponent of modal extra-mathematical explanation is Lange (2013), who points out that 23 strawberries cannot be evenly divided among 3 children, no matter what the physical laws of a possible world containing strawberries and children.

events, they are often vastly more complex than reflectance, which appears to be one of the most fundamental scientific properties. I am concerned that much extra-mathematical explanation transpires above this fundamental level where I see a conceptual regress lurking (chapter 2), since larger scientific systems often build from the smaller.

When theorists *do* debate how a mathematically-defined physical property can be instantiated, it is often in the context of idealizations, which I discuss in section 4.2. For the time being, it pays to flag one of the key insights of my argument: that the conditionalization of mathematical realism on property realism can in some cases reify mathematics *without* the EIA. As I argue in chapter 5, this insight has immediate relevance, since Knowles and Saatsi (2019, §1) find debate on the EIA to have "reached a serious impasse," and my insight about reflectance circumvents the impasse.

Although I cannot tell the entire impasse story here, Marcus (2015) offers an insightful critique of the EIA, which facilitates forthcoming discussion. Specifically, he criticizes EIA premise (1) for equivocating on what is meant by "explanation . . ." (138). He argues that explanatory power is *already* one of the virtues that the Quinean carefully considers in theory selection before regimenting and quantifying existents (126). Thus, when the EIA theorist appeals to an entity's "explanatory" role in premise (1), Marcus infers "explanatory" to take one of two senses: (i) a "metaphysical" sense that picks out ontology without regard for human understanding or comprehensibility, or (ii) an "epistemic" (137) sense that idealizes and abbreviates theory precisely *to* enhance human understanding rather than get ontology exactly right. Hence Marcus's dilemma for the EIA: either "explanatory" is type-(i) metaphysical, and so EIA differs not a whit from QI, or "explanatory" is type-(ii) epistemic, and so particularly *un*suitable for picking out mathematical existents!

While I find this argument thought-provoking, it is slightly too quick. For as I anticipated in section 3.2.2 and elaborate in chapter 5, the EIA admits as premises explanation forms that QI is tailored to reject. The "program explanation" of Frank Jackson and Philip Pettit (1990), for example, which Aidan Lyon (2012) turns into an extra-mathematical explanation, quantifies over real properties and purports to give them both ontic and epistemic explanatory power; but the first-order logic of QI notoriously shuffles properties out of the ontological deck (as "cross-fertile species" were eliminated in section 3.2.2). Thus, while quantifying over properties in an explanation is a "metaphysical" step, it is not necessarily a step of QI, and so horn (i) of Marcus's dilemma breaks off.[38]

Thus ends my review of EIA premise (1), generally construed. I now examine the controversy surrounding premise (2) ("Mathematical objects play an indispensable explanatory role in science"). While some theorists identify what I take to be a Marcusian type-(i) metaphysical explanatory role for mathematical entities,[39] others endorse a type-(ii) epistemic role,[40] and still others deny that extra-mathematical

[38] See my chapter 5 for how the epistemic horn (ii) of Marcus's dilemma for EIA might also break off, if extra-mathematical program explanations are epistemic explanations; although I will not critique Marcus's account further in this dissertation.

[39] Andersen (2018), Chirimuuta (2018), Berenstain (2017), Baron (2014), Lyon and Colyvan (2008), French (2006, §2) who discusses Fourier mathematics, Batterman (1997), and Franklin (1989).

[40] Bangu (2020), Shech (2019), Knowles and Saatsi (2019), Jansson and Saatsi (2019), Strevens (2018), Bokulich (2018) who discusses Fourier mathematics, Lange (2016, Chap. 11), Molinini (2016), Skow (2014), Pincock (2011), Dorato and Felline (2011) who discuss Fourier mathematics, Batterman (2010), and Leng (2005).

explanation succeeds in any sense whatsoever.[41,42] Nevertheless, as I disclaimed earlier in this section, I do not see that my thesis affords me an immediate stake in this debate about EIA premise (2). For in trying to explain how fundamental physical properties can be instantiated at all, I invariably fly under the radar of most of the extant discussion on EIA premise (2).

An example, not of EIA proper, but of the role of mathematics in scientific explanation, which differs significantly from my reflectance inquiry, is Alisa Bokulich's (2018) explanation of sand-ripples (regularly-spaced mounds of sand caused by air or water waves) using a Fourier-mathematical model. Firstly, her central focus is to elucidate why the model counts as a causal (and non-causal) explanation of the ripples and their characteristic wavelength, a distinction that I see no reason to draw when attempting to simply ascribe reflectance. Secondly, she (like practically all other interlocutors on EIA premise (2)) set out to explain an observable, something *to be seen*—a pattern of sand ripples,[43] whereas I find mathematics indispensable to making sense of the concept *that something could be seen at all*, specifically visible at all, or reflective. Thirdly, Bokulich omits to discuss the significance of the infinite space over which one of her explanantia sand-equations integrates (152), whereas the very crux of my argument is to exploit a similar infinitude to stop a vicious conceptual regress about reflectance ascription (chapter 2). Fourthly, whereas Bokulich's Fourier explanantia

[41] Brown (2012, Chapter 1), Bliss and Fernández (2010), Daly and Langford (2009), van Fraassen (2006, 285, n. 7), and Cao (2003).

[42] Of course, my taxonomy of these positions could be disputed, but altering the list does not affect my thesis.

[43] Other authors, as my later chapters detail, study observables like rainbow color spacings and angles, the shape of breaking drops, soap film geometry, and pedestrian bridge networks.

yield model outputs in "reasonably" good agreement with experiment (153), I am not after such agreement. I am trying to define the real property (reflectance) about which an experiment could even be invented and conducted. Sand-ripples are arguably not an "intrinsic" property of anything, but an exogenous confluence of sand "mass flux" at a given "bed elevation" and grain incidence angle, etc. (152). I am after something much simpler and more fundamental (property ascription), and because my pursuits are so characterized, some may suspect that I am needlessly resisting the thesis that some properties can only be discussed in idealized form, and that the harmonics whose conditional reality I defend just are idealizations. I elaborate and respond to this criticism next.

4.2 Explanation and Idealization

To give away my conclusion early, I deny that idealization is a good characterization of the Fourier harmonics indispensable to reflectance ascription, because of what I understand to be the fundamentality of reflectance within science. Fundamental properties are supposed to be those that we do *not* idealize on sight,[44] for they are fundamental and partially constitute larger systems. Hence fundamental properties are real if we are realists (see Chakravartty 1998), and they are accepted into theory but not fancifully invented if we are antirealists.[45] Each of my chapters grapples with the

[44] I see myself following Azzouni (2005, 56) in this regard, who treats fundamental *laws* as true simpliciter in his scientific realism. Ellis (1992) holds an opposite view epistemically, finding idealization essential to learning the natures of fundamental properties. I criticize Ellis's (1992) account of idealization and the Fourier harmonic in chapter 3.

[45] By the antirealist not "fancifully inventing" her empirically adequate theories, I have in mind Wray's (2018) focus on data-driven antirealism, and his natural selection analogy, by which he argues that scientific theories currently in circulation are non-miraculously successful because inferior competing theories have died off (148 ff.).

implications of reflectance realism, because most of the authors with whom I engage are property realists, so I hereafter leave property antirealism aside.

Nevertheless, when confronted with my appeal to harmonic realism for rendering reflectance conceptually coherent, interlocutors often respond—and reasonably so—by suggesting that I am likely *somehow* employing an infinite idealization. I continue to find significant discrepancies, however, between my reflectance example and the current leading accounts of idealization, discrepancies that I summarize in this section in a first-pass manner (since the topic of idealization is vast). That is, my dissertation takes no position on which accounts of idealization are best,[46] or how the practice should go generally. Hence, I aim in this section to create conceptual distance between my thesis and leading accounts of idealization, so that my dissertation chapters may be read for their metaphysical insights, tentatively freed from the constant threat that an act of idealization would scuttle the entire discussion.

To additionally flag a separate issue, I acknowledge that assuming infinitudes *to be* mathematical is by no means a foregone philosophical conclusion.[47] Whereas Azzouni (1994, 3) treats the infinite as essentially mathematical along with applied continua and infinitesimals (Azzouni 2009, 161), Field (1980, 95, 101-102) nominalizes infinitudes and references them in non-mathematical language, and René Guénon (2001) interprets the infinite as neither physical nor mathematical, but metaphysical, since it would literally encompass all things, as nothing could limit it. One straightforward assumption of my thesis is that I only conclude a limited mathematical realism (when I

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[46] I do criticize Ellis's (1992) method of *applying* idealization, in chapter 3.

[47] Neither is it a given that all mathematizations of theories and models are idealizations, according to Baron (2016).

do) because I assume with Azzouni that infinitude is a mathematical property. Even if I am wrong in this assumption, my dissertation still introduces *real* infinitudes or infinite structure into several philosophy of science debates, and some theorists would idealize that structure even if it were not mathematical. Thus, the following disclaimers about infinite idealization are dialectically important.

4.2.1 John Norton on Approximation and Idealization

In section 4.1, I disavowed at length any responsibility to construe my reflectance example as an explanation (or as an instance of EIA), and so I here footnote rather than analyze recent discussions on whether idealizations explain,[48] and whether *infinite* idealizations explain.[49,50] What I can more reasonably accommodate is the suggestion that my harmonic redefinition of reflectance is a model or template that real reflectance or reflection events approximate. Although I will deny this suggestion, let us follow the influential John Norton (2014) in defining "approximation" as "an inexact description of a target system. It is propositional" (199). An "idealization," on the other hand, "is a real or fictitious system, distinct from the target system, some of whose properties provide an inexact description of some aspects of the target system" (199).

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[48] Setting aside Marcus's (2015) metaphysical/epistemic distinction about explanation, those who think that idealizations explain include Andersen (2018, §5.2), Bueno and French (2018, Chap. 9), Potochnik (2017, Chapter 5), Shech (2015), Pincock (2011), Wayne (2011), and Ellis (1992); Baron (2016) and Strevens (2008, Chap. 8) deny, with qualifications, that idealizations explain.

[49] Setting aside Marcus's (2015) metaphysical/epistemic distinction about explanation, those who think that *infinite* idealizations explain include Shech (2019), Andersen (2018, §5.2), Pincock (2011), and Batterman (2005); Baron (2019) and Strevens (2019) deny that infinitudes explain, with qualifications.

[50] See Liu (2019, 1887) for the intriguing thesis that *all* idealizations are infinite idealizations, because idealizations tend to be quantitative, and so can always be brought infinitely or infinitesimally closer to exact representations. Liu rejects that strong thesis, but opines that "most idealizations are infinite idealizations . . ." (1906).

An accessible example of idealization in Norton's (2012, §3.1) account is his bisecting a sphere of unit radius and adjoining the sections to either side of a unit-radius cylinder of length *a* to form a "capsule." The ratio of surface area to volume (SAV) of real capsules approaches 2 as *a* goes to infinity, and the infinitely-long capsule *has* a SAV ratio of 2. This SAV correspondence between real and infinitely long capsules indicates that the infinitely long capsule is an idealization, and an informative one. The SAV ratio of 2 in the idealized capsule "inexactly describes" the SAV ratio to be expected in real capsules. When there *is no* limit system, on the other hand, but the real property in the target system approaches a limit value, then the limit property is an approximation of the properties of real targets (Norton 2012, §3.2). Norton's (2012, §3.2) example for approximation is the sphere, whose SAV ratio is 3/*r*; at infinite radius, the ratio goes to 0, but there *is no such thing*, Norton claims, as a set of points infinitely distanced from a center point (213); the infinite sphere is not as well-defined in space as the infinitely long capsule is, so 3/*r* only approximates the SAV values of real spheres.

The upshot is that there are a number of ways in which the infinite-duration harmonics that I posit for reflectance ascription are *neither* an idealization nor an approximation in the Nortonian sense. Whether or not there *is* a limit system of infinite-duration waves impinging upon and reflecting from a surface does not obviously concern me, because I am not trying to use that system to learn anything about finite-duration properties or events analogous to the SAV ratios of real capsules. I am not wondering what reflectance *is* (or how pulses behave) in a long-duration limit, or even in a short-duration limit. I instead seek to understand how "intrinsic" dispositional reflectance can be ascribed using pulses of any duration at all (the received view), and the cleanest and most expeditious answer I find is that “intrinsic” per-wavelength reflectance can only be the disposition to reflect infinite-duration harmonics; defining reflectance with finite-duration radiation generates conceptual regress (chapter 2). I do not see Norton attempting to avoid regress, and so I do not see what benefit I gain by taking idealization and approximation onboard my inquiry. Secondly, one should pause to consider how strange it is to suggest that “intrinsic” reflectance could have distinct “idealized” and “target” models or formulations. For if “intrinsic” roughly means “by itself” or “due only to itself,” how could something be *ideally* “by itself” in a way that differed from its *actually* being “by itself”? In positing intrinsic reflectance, I seem to have no need of both an idealized and target system (a point that I resume in chapter 3).

Neither, in my view, is my positing of harmonics a Nortonian approximation, or a standard against which to formulate approximations. It is not the case (as I see it) that real reflectance properties approach a limit in their ascribability[51] as the durations of their stimuli and manifestations approach infinity, because I take ascribability to be all-or-nothing (see chapter 2, section 4). By all-or-nothing, I mean that surfaces are either reflective or not; when the duration of light makes-or-breaks a surface’s *possession* of reflectance, it is no longer *surface* reflectance, but a combination property of surface-and-light. Perhaps some surfaces require a minimum incident average power before they will reflect any light, but this qualification is moot with respect to defining without regress what reflectance (when it does exist) *is*. I acknowledge that these disclaimers in advance of my detailing the reflectance regress in chapter 2 may appear obscure, but my point

[51] Per my discussion of section 1, “ascribability” just means conceptual coherence and lack of conceptual regress.

(again) is to hold idealization objections at arm's length, since although they may appear to apply in every chapter, I keep finding reasons to doubt that they do apply. I give some such defenses in context, when they arise.

4.2.2 Derivatives and Alternatives to Norton's Account of Idealization

Another influential account of idealization is Michael Strevens's. He argues that the function of idealization is to indicate how *little* some parameter (in my case, pulse duration) matters causally to a given analysandum (Strevens 2008, 315), or how little a given parameter matters to a causal or non-causal explanation (Strevens 2019, 1715; 2008, Chap. 8). Air resistance, for example, does not matter to the explanation of why a pendulum with constant arm-length swings at a constant period invariant with bob mass, so the idealized equation of pendulum motion omits air resistance. As I have now remarked several times, however, my reflectance example does not purport to explain any physical phenomenon or identify causes. More to the point, my infinite-duration harmonics are *the furthest thing from* a Strevens idealization, because according to my argument (chapter 2), harmonics make-or-break the property ascription; instead of mattering not-at-all, harmonics matter entirely.

While I denied in section 4.2.1 that my reflectance example well fit Norton's account of idealization and approximation, some may have recognized by now that my example *exactly* corresponds to a derivative of Norton's account, namely Elay Shech's (2015). What he calls "an *essential* idealization" is an idealization in the Nortonian sense, except that "the property of the limit system does not correspond to the limit property in the sense that any de-idealization from the limit systems renders the property nonexistent" (1069). This remark describes exactly what happens in the reflectance regress that I generate in nearly every chapter of this dissertation, but two insights prevent me from calling my use of harmonics an "essential idealization" in Shech's sense. The first, as I have said, is that no one suspects that the very *ascription* of reflectance is an ideal system! Reflectance is supposed to be more basic than all systems; it is the property by which we interact with practically any entity, at any scale (see chapter 6).

Secondly, Shech's (2015) application of essential idealization arises in the context of the Aharonov–Bohm (A-B) effect, for which he needs the ideal system known as a non-simply connected (or multiply connected) topological space with non-trivial fundamental group; this property represents the infinitely-long solenoid of shielded wire around which the A-B effect emerges. Even granting, however, that some (controversially) consider the A-B effect to be as real as reflectance, I object (again) that reflectance is not an *effect*, it is a *property*. I am doing metaphysics before I am engaging in scientific description or explanation, and so I see my project tangential to Shech's and Bokulich's (section 4.1) in a similar regard.

Some may dislike my continual reflectance-is-not-a-phenomenon dodge, and insist that reflectance (unexpectedly and disappointingly) joins the fray of other physical realities in need of asymptotic or otherwise infinite idealization for insertion into theories and models. That is, while my reader may understand well that I need not employ idealizations for theoretical tractability (Pincock 2014; Strevens 2019), or for interpreting other idealizations (Pincock 2011), some may nevertheless suspect that I need harmonics in the way that theorists need an infinite number of water molecules to reduce thermodynamics to statistical mechanics in a phase transition, a much-discussed example[52] with relevant analogs.[53] My general reply to this contention is that the examples listed in the previous sentence and associated footnotes are extremely complicated, and that before an analysis of that order is justified, I think that the reflectance example needs to be tried against first-order accounts of property ascription, explanation, and the role of mathematics in science. Hence my subsequent chapters.

5. My Overview of Chapters

While I have hinted at the contents of some chapters in the preceding, I summarize all of them now, for thoroughness. Chapter 2 deploys the most mathematics of any of my chapters, to support my central reflectance argument: that the received definition of reflectance as the disposition to reflect finite-duration "pulses" of light suffers conceptual regress and needs redefinition in terms of the Fourier harmonics that superimpose into pulses. Those comfortable with the regress argument of chapter 2, section 3, may skip over its restatement in subsequent chapters, which is usually clearly marked by section title. While chapter 2 remains neutral on harmonic realism, it sets the stage for other chapters to generate contingent realism claims.

Chapter 3 ventures my first harmonic realism claim, contingent on the dispositional and structural realism comprising Anjan Chakravartty's (2007; 1998) scientific "semirealism." The semirealist construes dispositions as intrinsic properties of

[52] E.g., Liu (2019), Baron (2019a), Shech (2018; 2013), Ardourel (2018), Norton (2012, §5), and Batterman (2010, §3; 2005, §§2-4).

[53] Including the reduction of Maxwellian optics to ray theory to explain rainbow properties (Pincock 2011), and the use of asymptotes in explaining the geometry of breaking drops (Batterman 2005). Particularly interesting about the breaking-drop example is that a "hydrodynamic description" of the drop fails to "be self-consistent" (Batterman 2005, 239), akin to how my Maxwellian description of reflectance becomes self-inconsistent (see my chapter 2).

entities, the manifestations of said dispositions being real structures described by the mathematical laws that survive theory change. One set of laws that Chakravartty analyzes is Augustin-Jean Fresnel's laws of refraction, and I introduce into the semirealist debate the insight that Fresnel's laws are the manifestation of dispositional refractivity. Dispositional refractivity, however, suffers a conceptual regress akin to the reflectance regress, and so I put to the semirealist a dilemma: either refractivity is not "intrinsic" to entities (because not manifesting infinite-duration harmonic structure), or not a "disposition," since a disposition typically "awaits" a stimulus in order to manifest, and is not *always* manifesting as the infinite-duration harmonic structure of "intrinsic" refractivity entails. I contrast refractivity with the dispositional solubility of salt: salt always *could* dissolve in the right quantity of water, but is not always dissolving. I additionally point out that Fresnel himself employed infinite temporal structure in his wave theory, but did not likely consider it ontologically real. The semirealist tenets of intrinsicality and dispositionality elevate, in my view, the ontological status of this temporal structure that Fresnel recognized.

Chapter 4 conditions harmonic realism on reflectance realism, but only according to the specific theoretic machinery that Jody Azzouni calls "thick epistemic access" (TEA). TEA is Azzouni's quaternary sufficient condition for the existence of posits referenced by terms in our scientific language, and I argue that the Fourier harmonic satisfies the fourth TEA requirement called **Grounding**. Posits play a **grounding** role when they or their properties "explain" (Azzouni 2004b, 383) how we are able to interact with the posit. The density of heart tissue, for example,[54] explains why the heart can be

[54] This example is Mark Colyvan's (2010), see my chapter 4.

distinguished from bones and lungs on an x-ray screen; density grounds our epistemic access to the heart, and so we should say that hearts exist. I reiterate this observation, arguing that sometimes a posit **grounds** our TEA to *another* posit. The Fourier harmonic, for example, makes reflectance ascribable (by blocking the reflectance regress), and so makes a vase reflective and visible, **grounding** human visual access to the vase. While I do not claim that the harmonic satisfies all four TEA criteria, I argue that a posit's satisfaction of the **grounding** condition suffices to garner ontological standing for that posit within Azzouni's account. I conclude that Fourier harmonics exist despite Azzouni's mathematical nominalism.

My strongest conditional claim for harmonic realism transpires in chapter 5, where I use the mere premise of property realism within Jackson and Pettit's (1990) "program explanation" methodology, to conclude harmonic realism from reflectance physicalism. I argue that dispositional reflectance is an explanans property in Jackson's (1998a) program explanation for human color experience, and I deploy this insight as a counterexample to Saatsi's (2016; 2012) criticisms of extra-mathematical program explanation (cf. section 4.1). Chapter 6, on the other hand, stops short of concluding harmonic realism, but shows harmonics to mimic the otherwise implausible role of homogenization within a multiscale model. Homogenizations are mathematical shortcuts—often blowups to infinity—used to communicate information between the scales of a model without referring to anything real. I contend that by ascribing reflectance, I am not attempting to communicate between scales, but rather to ascribe a property ostensibly univocal (because fundamental) at all scales. Hence the multiscale modeler should explain the indispensable role that I identify for the harmonic in reflectance ascription, since ignoring the harmonic and adopting the received view of reflectance generates a meta-epistemic “hurdle” in multiscale modeling: the attempt to articulate the scale-dependent properties of certain materials (nanoparticles, microtubules, etc.) *by way of* manipulating and measuring a different property with its own regressive incoherence at all timescales (reflectance).

Chapter 7 returns to the theme of mathematical nominalism broached in chapter 4. Against Field’s conservativeness thesis about mathematics (cf. section 3.1.3 above), I argue that Fourier analysis is nonconservative over physical theory, since adding Fourier language to a contrived nominalist language N* about the average powers of electromagnetic pulses yields new nominalist sentences not derivable from N* alone. These new sentences include, “This surface is reflective,” and “Surface A is more reflective than surface B.” I argue that the per-wavelength reflectance regress launches within N*, and so makes the novel sentences nonsensical in N*.

Lastly, because the account of reflectance that I analyze has its origins as a reduction base for color ontology in the philosophy of perception, I assume in chapter 8 that such a reduction to my redefined harmonic-reflectance holds, and I examine implications of that reduction for theological aesthetics—a significant step away from the philosophy of science, but a justified one, considering the reflectance subject matter.[55] One implication of my color reduction is support for Christopher Longhurst’s (2012) *Via negativa* analysis of black monochrome abstract paintings. Whereas he perceives in

[55] Indeed, early in my philosophical studies of reflectance, I had to learn how to convince my interlocutors right away that I was *not* talking about perceptual theory, since the association in many philosophers’ minds between reflectance and color ontology is so strong.

those paintings infinite spatial dimensions that facilitate the viewer's encounter with the divine, I add that if the color black reduces ontologically to harmonic-reflectance, then the color of those monochromes introduces an additional infinite dimension: a temporal one, in which the viewer is equally absorbed. A second implication of my color reduction is to undermine Paul Crowther's (2016) metaphysics of human creativity, by which human artists bring space and time themselves into being. This metaphysics is incompatible with my color reduction, I argue, since for the color reduction base of reflectance to be conceptually coherent, its manifestation must transcend the artist. Chapter 9 concludes with one unconsidered objection to my thesis, and suggests one direction in which a possible elaboration from an idea in chapter 4 might lead.

CHAPTER 2

HOW TO MAKE REFLECTANCE A SURFACE PROPERTY*

One of the commonest properties in science and everyday experience is ‘reflectance’, the property of car doors, mirrors, and luxury fountain pens that allows you to see your face in them (and radar systems to detect them, for that matter). Philosophers like David R. Hilbert (1987) define this property as Surface Spectral Reflectance (SSR),[1] and in this essay I contend that SSR as currently defined is *not* a surface property, but a combination property of surface-and-medium or surface-and-light.[2] What I mean by calling Hilbertian SSR a surface-and-medium property is that Hilbertian SSR is an *extrinsic disposition* (McKitrick 2003) of surfaces, contrary to Hilbert’s (1987) claim that SSR “is an intrinsic, illumination-independent” (1177) disposition.[3] To deny that Hilbertian reflectance is

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* This chapter is the dissertation-editorialized Accepted Version of Nicholas Danne, “How to make reflectance a surface property,” *Studies in History and Philosophy of Modern Physics* 70 (2020): 19-27, https://doi.org/10.1016/j.shpsb.2020.01.002.

[1] Hilbert continues to defend the concept of SSR as a surface property with Alex Byrne (Byrne and Hilbert 2003, 2004, 2007). Recent philosophers writing on SSR include Jackson (1998), Chirimuuta (2015), Gert (2017), and Isaac (2018).

[2] Any mention of “light” in this essay could be replaced with terms referring to alternative portions of the electromagnetic spectrum, such as radar or commercial radio. I argue in terms of light because Hilbert does, and because of an important problem for lased light that I explain in later sections.

[3] Hilbert (1987) calls SSR a ‘disposition’ at 386, 499, 1036, 1176, 1749, 2086 (Kindle Locations), and at Byrne and Hilbert 2003, p. 20, endnote 13. Jackson (1998) also understands SSR to be a disposition, but as supervening on a to-be-determined categorical base property (e.g. surface microstructure). I return to this categorical/dispositional distinction in the next section. Otherwise, Hilbert generally declines to defend a specific metaphysics of dispositions (tropes, universals), or to formalize the stimulus and manifestation of SSR, and so I follow suit.

intrinsic will strike some as unintuitive (e.g. Isacc 2018, 521, 524; Byrne and Hilbert 2003): mirrors are supposed to remain 'reflective' objects even in the dark, just as a vase remains 'fragile' even if never dropped. By calling Hilbertian SSR an extrinsic disposition, however, I mean that light partially *bestows upon* mirrors their (Hilbertian) reflective capacity or reflectance profile. To the degree that this metaphysic of reflectance seems wrong, then Hilbert's definition requires what I show in this paper to be a somewhat mathematized reworking.

Understanding my argument requires first understanding what Hilbert means by intrinsic. He does not precisely define the term, but his account accommodates the intuitions outlined by Jennifer McKitrick (2003):

> Intuitively, a property is intrinsic if anything that has it has it regardless of what is going on outside of itself. . . . Extrinsic properties, by contrast, are simply those that are not intrinsic. If a property is extrinsic, it is possible that a thing's having that property depends on what is going on outside of the thing. (McKitrick 2003, 158)

The surface of a large airplane, for example, may be considered to be its paint coat, and that paint coat can possess a *weight* of hundreds of pounds. But weight is an extrinsic disposition of the paint, because weight obtains and takes its definition indispensably from the terrestrial mass on which the airplane rests (e.g. Earth, Mars, the moon) (McKitrick 2003, 160). The mass of the airplane paint, on the other hand, is intrinsic to that paint (McKitrick 2003, 160), and so I would call mass but not weight a surface property obtaining at the airplane's paint coat.[4] Paint weight is not a surface property, but a surface-and-planet property.

[4] I ignore the complication that a paint coat may or may not itself have a 'surface'. Hilbert gives no metaphysic of surfaces, and so neither shall I.

By a similar but more complex argument, I claim that Hilbertian SSR cannot be an intrinsic disposition, because Hilbert defines SSR as the disposition to reflect *pulses* of light. Pulses are finite-duration propagations of electromagnetic radiation, and I object that Hilbert's definition fails to account for a well-documented, empirical phenomenon of pulses that I call 'harmonic dispersion'. Harmonic dispersion is the inverse relationship of a pulse's duration to its bandwidth, and from this premise I argue that at short pulse durations, pulse-SSR (Hilbertian SSR) becomes *undefined*. More specifically, pulse-SSR is undefined metaphysically[5] at all pulse durations, because pulse-SSR suffers a vicious infinite regress of conceptual incoherence (section 3); but because any number of *operational* definitions may fall short of 'well-definedness' variously construed (e.g. the circular definition of temperature as what the thermometer says), I additionally argue the second and distinct thesis that Hilbertian SSR is not even *operationally* intrinsic, or intrinsic *qua* useable by practitioners who profitably ignore its conceptual incoherence. Granted, I do not deny that pulse-reflectance theorists succeed at their discipline and technological development; I only deny that pulse-reflectance is intrinsic to surfaces.

More positively, I argue that the disposition to reflect the *harmonic components* of a given pulse, components that by definition do not exhibit harmonic dispersion (see section 2), can be an intrinsic disposition in both the metaphysical and operational senses. Because a number of philosophers and scientists would reject an ontology of (Fourier) harmonics as entities literally reflecting from surfaces (Wilson 2017, Thalos 2013, McGivern 2008, Liston 1994, Redhead 1988, cf. Sheldon 1985), however, my

[5] By the "metaphysical" definition of a property, I mean in the very weak sense a definition free of vicious conceptual regress. I present an example of such a regress in section 3.

redefinition of SSR proves non-trivial.[6] I remain neutral on the question of mathematical realism (i.e. harmonic realism) in this paper, but I implicitly show that the intrinsicality requirement for reflectance brings the question to the fore.

My argument proceeds in five steps. Section 1 reviews Hilbert's employment of SSR and the tacit but consequential metaphysical assumptions that he adopts in defining it. Section 2 illustrates the concepts examined in section 1, providing a short tutorial on Fourier analysis. Section 3 models the harmonic dispersion of a reflecting pulse to reveal why Hilbertian SSR is not a surface property, and the final two sections answer objections and conclude the paper.

1. How Hilbert Defines and Employs SSR

The first point to acknowledge is that Hilbert employs SSR in a theory of *color perception*. That is, he defines colors as *sets* of reflectances. Hence what I am arguing is that each individual SSR disposition that partially comprises one of Hilbert's reflectance sets cannot be a surface property, and so neither can Hilbertian surface color. Because I have nothing to say about perceptual theory, however, I focus my discussion on the individual SSR dispositions that Hilbert employs, since these dispositions are ostensibly the same kind of 'reflectance' that obtains in non-color sciences like laser physics (Stingl et al. 1995), geophysics (Gaffey 1976), radar (Haykin 1989), and others. If intrinsic pulse-reflectance is undefined in one branch of science, in other words, then I consider it undefined for any and all branches of science.

[6] As for why I decompose the pulse according to a Fourier basis instead of by some other basis such as wavelets, see section 4, footnote 35.

My evidence that Hilbert (1987) likewise thinks that SSR or 'reflectance' is of a univocal kind in the sciences is the following quote:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987, 1037-1041)

Thus SSR is the per-wavelength efficiency of a surface to reflect light "flux." By "flux," Hilbert means the power (in Watts) of incident or reflected light (Hilbert 1987, 1033-1042), and because light propagates sinusoidally (or as a modulated function of sine), flux is the average, as opposed to instantaneous power of propagating light.[7] But here marks the first of Hilbert's very consequential maneuvers. Equating flux with average power follows the standard terminology of spectrophotometry (Germer et al. 2014),[8] but electromagnetic signal theory interprets "average power" differently. In the remainder of this section, I explain why this difference matters to the philosophy of dispositional property ascription and of reflectance in particular.

[7] Said another way, the instantaneous power of a signal is a function of its instantaneous amplitude, but the instantaneous amplitude of a propagating wave changes with time and spatial location, so usually only the average power is a useful quantity. That some disciplines use hyperbolic secant or Gaussian pulses instead of sinusoidal pulses, to model the non-instantaneous ramp-up and decay of real pulse amplitudes, does not affect my argument, since pulses of all such shapes exhibit harmonic dispersion. Hence the power spectra of Gaussian and hyperbolic secant pulses resemble well enough the spectrum of a sinusoidal pulse that I use to generate the vicious conceptual regress of SSR (sections 2 and 3); for the power spectrum is a tool routinely utilized by the laser sciences to analyze electromagnetic signals (Stingl et al. 1995). I thank an anonymous reviewer for pressing me to justify this methodology in my paper.

[8] Spectrophotometry is one method of measuring SSR. A spectrophotometer measures the average power of light reflected from an object, by measuring electrical currents conducted by photoresistors sensitive to the optical bandwidths of interest.

According to signal theory, the only signals that possess average power are called "power signals," because power signals have infinite duration over which to integrate their "infinite energy"; signals with finite duration are called "energy signals" and have zero average power, because their average power, too, is computed by an integration over infinite time (Haykin and Van Veen 1999, 20-21). Thus 'energy signal' and 'power signal' are signal-theoretic *names* for finite-duration and infinite-duration signals, respectively. With respect to my thesis, why should any SSR theorist or spectrophotometrist care about *infinite* durations? No light shines on a surface for infinite duration, and if our universe originated as a Big Bang, then presumably no light *could* propagate to a surface for infinite duration. Hilbert and the spectrophotometrists have dispensed with signal theory to employ a shorthand version of 'average power' for finite-duration signals, and this shorthand suits their operational purposes (namely, measuring reflectance with *hyper-picosecond* pulses[9]). For even the very lean metaphysics of Hilbertian SSR, however, this colloquialism proves devastating.

[9] Light pulses disperse high-amplitude harmonics further than 1 nm from the pulse's fundamental wavelength (i.e. "center frequency" or "carrier frequency") when pulse duration falls below approximately 1 picosecond (ps, 10^{-12} seconds). Section 3 illustrates this phenomenon that I have already dubbed 'harmonic dispersion'. An anonymous reviewer asks why I make '1 ps' the threshold for metaphysically problematic dispersion, if harmonic dispersion ostensibly infects *all* finite-duration pulses. I answer that I choose the 1 nm/1 ps threshold because some applications of reflectance within Hilbertian color theory count a 1 nm discrepancy as moderate to severe error (e.g. Gert 2017, 69). Other, non-color applications tolerate more or less error than 1 nm, and so the argument of this paper could be run for wider or narrower bandwidth dispersions, per application. But the point is not that color science proceeds in the sub-picosecond domain; on the contrary, Scase and Foster (1988, 196) report loss of hue discrimination in humans for exposure times below a few milliseconds. The point is that pulse-SSR conceived operationally fails when moving from color applications to sub-picosecond laser applications, and because this operational success or failure depends on pulse duration, pulse-SSR is operationally an extrinsic disposition (see section 3). That pulse-SSR also fails to be *metaphysically* intrinsic to surfaces, I argue in section 4.

I claim that the SSR metaphysician needs infinite-duration power signals for the same reason that signal theorists do: because of the indispensability[10] of power signals in accounting for harmonic dispersion (e.g. Haykin 1989, 36-37, on radar applications). I explain and illustrate harmonic dispersion in detail in the next section, but for now I need to outline the problem that harmonic dispersion creates for calling Hilbertian SSR a surface property.

Per the introduction in the previous section, harmonic dispersion is the tendency of an electromagnetic pulse, which propagates at a dominant carrier or center frequency, to propagate at more than one frequency when pulse durations become extremely short.[11] This tendency, moreover, is not some kind of 'quantum problem' misapplied to the medium-sized dry goods of macroscopic science. The phenomenon is neither measurement noise nor an artifact of instrumentation. Harmonic dispersion is a principle of classical, Maxwellian electromagnetics (Hirlimann 2005, 31; Haykin 1989, 36-37). Thus because some long-duration pulses consist predominantly of *one* wavelength, but thanks to harmonic dispersion *all* short-duration pulses propagate as if composed of *multiple* wavelength components in superposition (section 2), the per-wavelength efficiency of a surface to reflect *pulses* (Hilbertian SSR) requires an application-

[10] While Fourier analysis may be supplanted by e.g. wavelet theory, I second Liston's (1993) implicit denial that harmonic dispersion could be theoretically formulated without mathematics. See footnote 35 of this paper for further discussion of alternatives to Fourier analysis.

[11] "Extreme" is a relative notion here. Light pulses propagate at hundreds of terahertz (THz, 10^{12} Hz) and disperse "radically" with respect to the goals of some scientific applications at durations below one picosecond. Radio waves with kilohertz carrier frequencies, on the other hand, radically disperse at millisecond durations. There is thus a billion-fold difference in what pulse durations count as "extremely" short across common electromagnetic applications.

dependent minimum, non-dispersive pulse duration in order to be *operationally* defined (section 3).[12] I therefore call Hilbertian SSR an extrinsic disposition, a surface-and-medium property instead of a surface property, because Hilbertian SSR requires for even its operational definition a specific minimum duration of the electromagnetic medium, something "going on outside of" the surface (McKitrick 2003).[13] More strongly, I argue that pulse-SSR (Hilbertian SSR) is metaphysically undefined at *all* durations due to its colloquial appeal to 'average power' (section 3), and that calling a metaphysically undefined property an intrinsic property is implausible (section 4).

These conclusions problematize Hilbert's metaphysic of SSR for several reasons. Firstly, because they directly contradict his later work. Byrne and Hilbert (2003) claim, for example, that "the relevant physical property [to which they reduce color] must be a property of objects (more strictly, *surfaces*)" (9, emphasis mine). They also "rule[] out" for their color reduction the "properties an object has only if it is actually reflecting light of a specific character – for instance, light with a certain wavelength-energy distribution (spectral power distribution), or wavelength composition" (9). My analysis in this paper reveals that the requirement of a minimum pulse duration above which Hilbertian SSR can be operationally defined *just is* the requirement for a certain "wavelength composition": namely a composition sufficiently narrow in bandwidth to enable the per-

[12] "Operationally defined" means operationally exploitable, like temperature defined as what the thermometer says; operationally exploitable dispositions might not be metaphysically well-defined, and pulse-SSR is one such example. Note that a pulse duration of zero does not render SSR undefined, but only unstimulated (Hilbert 1987, Chapter 3).

[13] To reduce pulse duration *to* the duration of the surface, in hopes of recapturing the intrinsicality of SSR, would be both unprecedented and unmotivated in contemporary science. Scientists routinely change pulse duration without recording that the *duration of a surface* changed, unless they are recording e.g. a chemical reaction *at* an aging surface.

wavelength reflectance *of pulses*. (And if Byrne and Hilbert 2003 are *not* discussing a merely operational definition, then so much the worse for them, due to my arguments in section 4 against calling a metaphysically undefined property intrinsic.)

As quoted in the introduction, furthermore, to define SSR as anything but a surface property also contradicts Hilbert's (1987) earlier work. In his endeavor to reduce color to sets of SSR dispositions, Hilbert rejects what he calls the "wavelength conception of color" (Hilbert 1987, 187; hereafter WCC), or any theory that correlates color perceptions to properties of electromagnetic radiation instead of the properties of surfaces. Of course, as a color theorist, Hilbert rejects WCC for some reasons outside the scope of this paper. He reviews certain experiments, for example, that according to him undermine the case for correlating *human color perceptions* to properties of the electromagnetic medium (e.g. Hilbert 1987, Chapter 4). Nevertheless, because Hilbertian colors are *sets* of the SSR dispositions that allegedly count as 'reflectance' in all other branches of science (see my argument in the second paragraph of this section), then *all of science* needs to reject WCC when defining (or accepting from philosophers the concept of) dispositional reflectance. The anti-WCC constraint just is Hilbert's stipulation that SSR be intrinsic to surfaces.

Lastly, other SSR theorists who sympathize with Hilbert's color reductionism, but who argue that the categorical base of dispositional surface color is possibly (pending empirical evidence) surface microstructure (Jackson 1998, 94), will not be indifferent to the conclusion that Hilbertian SSR is not a surface property, but a surface-and-medium property. For how could reflectance-*qua*-microstructure be a shared or joint property of the electromagnetic medium? The medium contains no microstructure!

Taking stock, I have just qualitatively argued (or promised to show) that Hilbertian SSR is not a surface property, due to the pervasiveness of harmonic dispersion among propagating pulses in the sciences. I now detail the relationship of harmonic dispersion to power signals (infinite duration) and energy signals (finite duration) in the next section, to prepare my argument in section 3 that Hilbertian SSR (defined in terms of energy signals) becomes operationally undefined at short durations, and metaphysically undefined at all durations. Readers familiar with Fourier methods may profitably skip to section 3.

2. Depicting Harmonic Dispersion, and Anticipating its Consequences

Recall that harmonic dispersion is the widely-documented[14] and empirically demonstrated inverse relationship of the duration of an electromagnetic pulse to its spectral bandwidth. Hence a preliminary step to understanding my challenge to Hilbert's concept of SSR as the *intrinsic* per-wavelength efficiency of a surface to reflect *pulses*, is to understand what kind of signals possess wavelengths. As already mentioned, such signals may be classified as energy signals (finite duration) and power signals (infinite duration).

[14] E.g. Hirlimann (2005); Stingl et al. (1995); Haykin (1989); Hardin (1988, 206, endnote 36). Hardin (1988) is the only philosopher of reflectance or color perception that I have seen acknowledge the existence of harmonic dispersion, although he does not employ the datum as I do in this paper.

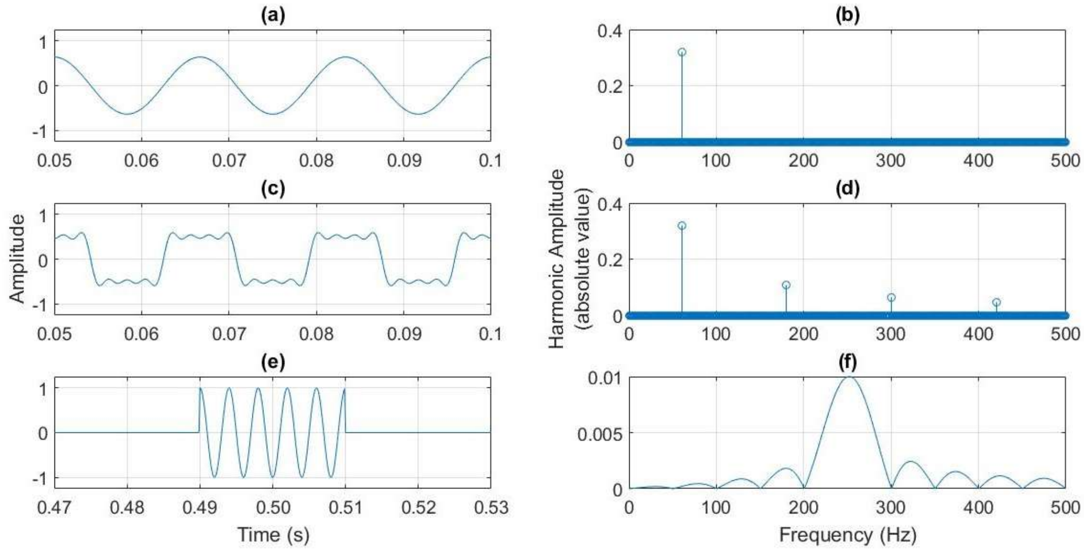

Figure 2.1: Superposition in the Time and Frequency Domains

Graphs (a)-(d) in Figure 2.1, for example, depict the power signals that superimpose to a (very rough) square wave. Graph (a) shows a 50 ms portion of a power signal with 60 Hz fundamental frequency. The units of amplitude do not matter, as they can be either electric or magnetic field strengths. Of crucial importance is the assumption that graph (a) reveals only a portion of an electromagnetic signal that possesses infinite duration. The power signal in graph (a) propagates infinitely after 0.1 seconds, and infinitely before 0.05 seconds, and so does not "start" propagating at t = 0 or t = 0.05; it has always been propagating.

The importance of the assumption that power signal (a) possesses infinite duration is that such an assumption renders power signal (a) the summation of impulses or "harmonics" in the frequency domain, as depicted in graph (b). Graph (b) reveals that power signal (a) is comprised simply of itself in the frequency domain: as a single, 60 Hz power signal. The 60 Hz signal in (b) superimposes with literally nothing else to become the power signal in (a).[15] One advantage of such Fourier analysis (decomposition) is that it allows signal theorists to identify which frequency-domain harmonics (be they physically real or not) contribute most strongly to the time domain signal of interest. In the case of square waves, graph (d) reveals that the additional superposition of 3rd, 5th, and 7th harmonics (at 180 Hz, 300 Hz, and 420 Hz, respectively) change the sinusoid in (a) to a much more suggestively square signature in (c).[16] Superimposing odd harmonics through the 29th or 99th index (not shown) renders the square wave very sharp indeed, with virtually no discernible ripple at the peaks (Haykin and Van Veen 1999, 178-179).

To recapitulate, graphs (a)-(d) in Figure 2.1 illustrate the time-domain and frequency-domain representations of *power* signals, which have infinite duration. Graphs (e) and (f) in Figure 2.1, on the other hand, depict the radically different spectral profiles possessed by *energy* signals, which have finite duration. The sinusoidal pulse in (e) is an energy signal, because its amplitude is zero for all times outside the window of $t = 0.49$ to $t = 0.51$ seconds (wherein its fundamental frequency is 250 Hz). Signal (e) does not propagate for eternity, and so it is not a power signal (although (e) is the *superposition* of the power signals in (f)). As a consequence, the frequency-domain equivalent of (e) is not a set of discrete impulses like (b) and (d), but a continuous function of harmonic

[15] More accurately, the 60 Hz harmonic superimposes with a -60 Hz harmonic (not shown). For clarity and transparency of applied mathematical assumptions, I omit negative-frequency harmonics from plots and break with the convention of adding their amplitudes to the plotted positive-frequency amplitudes. If someone should object that my argument lends ontological standing to negative-frequency signals, then so be it. I am not defending an ontology of mathematics in this paper, but only showing the implications of calling the reflectance disposition a surface property.

[16] Not every odd harmonic (3 through 7) is *added* to create the plot in (c). Some harmonics have negative amplitudes and therefore subtract from the superposition, but plots (b), (d), and (f) graph only the absolute values of harmonics, for aesthetic purposes.

frequencies called a *sinc* function (graph (f)).[17] This difference between the discrete and continuous spectra of infinite and finite-duration signals proves important for my thesis. For according to graph (f), *sinc* function spectra exhibit peak amplitude at the fundamental frequency of the (e)-pulse (250 Hz), but disperse additional, non-fundamental harmonics (e.g. at 175 Hz and 325 Hz) in proportion to the *brevity* or shortness of the duration of the energy signal (e). Plots (a) through (f) illustrate, in other words, the inverse relationship of harmonic dispersion: short pulses have wide spectral bandwidth, and infinitely long pulses have single-frequency or unity bandwidth.

It would be a mistake, moreover, to consider the preceding a sheerly theoretical exercise. The harmonic dispersion depicted in Figure 2.1 (f) finds practical application in the design of lasers (Hirlimann 2005; Stingl et al. 1995). Consider the remark by Stingl et al. (1995), for example, who identify "[t]he extremely broad bandwidth *necessary to generate* electromagnetic energy in such short intervals" (602, emphasis mine), "short intervals" being pulse durations below 10 femtoseconds (fs; 10^{-15} seconds). The wide bandwidth of short pulses is a mainstay of physical optics.

Similarly important to reject is the insinuation mentioned in section 1, that harmonic dispersion (depicted in (f)) amounts to "measurement noise" or imprecision of instruments. Firstly, plots (e) and (f) are not field screen-shots, but are the input and output, respectively, of a mathematical derivation of the spectrum of a perfectly clean (noise-free, distortion-free) sinusoidal pulse. Secondly, harmonic dispersion *in nature* or in the laboratory is not noise, or any imperfect divergence of physical signals from

[17] Haykin (1989, 18) defines $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$. For the *sinc* function in Figure 2.1 (f), x in the equation here given represents frequency in Hz.

theoretical signals. On the contrary, Stingl et al. (1995) find harmonic dispersion a target of fruitful "[e]xploitation" in laser science (602). That fiber-optics engineers try to compensate for, or offset various kinds of pulse dispersion in practice does not entail that harmonic dispersion is noise that Hilbert *qua* metaphysician (or alternatively, *qua* operationalist) can ignore in calling SSR intrinsic.

The purpose of this section has been to depict graphically what it means for a signal to reflect 'per wavelength'. While infinite-duration signals reflect exclusively at one wavelength, finite-duration signals (i.e. *pulses*) always reflect across an envelope of wavelengths. With this observation in hand, the metaphysics of SSR ascription can be better understood and problematized. More specifically, it can be shown that pulse-reflectance is operationally undefined for dispersive, short-durations, and metaphysically undefined at all durations. I run that argument in the next section.

3. Why Energy-Signal Reflectance (Hilbertian SSR) Cannot be a Surface Property

I shall now attempt to demonstrate how Hilbertian SSR (energy-signal reflectance) becomes operationally undefined at short pulse durations, and metaphysically undefined at all pulse durations.[18] The *sinc* function that represents energy signals[19] in the frequency domain, assuming time-domain amplitude A, duration T, and fundamental or carrier frequency f_c is:

$$G(f) = \frac{AT}{2}\{\mathrm{sinc}[T(f - f_c)] + \mathrm{sinc}[T(f + f_c)]\} \qquad [1]^{20}$$

[18] The results of the computer modeling that I undertake in this section closely match the physical results of laser experiments like Hirlimann (2005, 31) and Stingl et al. (1995).

[19] Like Figure 2.1 (e), in section 2.

[20] Equation [1] appears (with different numbering) in Haykin (1989, 37). Haykin (1989, 18) defines $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$. Per footnote 15 above, I omit to plot the negative-frequency "sinc[$T(f+f_c)$]" component of [1] for convenience of exposition.

Let f_c = 462 THz represent the fundamental frequency of a pulse of light with wavelength 650 nm. Fixing the duration of the pulse to T = 10 femtoseconds (fs; 10^{-15} seconds), and the amplitude to A = 1e12 ($1*10^{12}$, or one trillion), permits graphic comparison with a second signal of the same frequency, A = 1e11, and T = 100 fs.[21] Figure 2.2 plots the positive-frequency component of [1] across a set of wavelengths utilized in laser science (Hirlimann 2005; Stingl et al. 1995), coincidentally the set of wavelengths to which human retinas are sensitive.

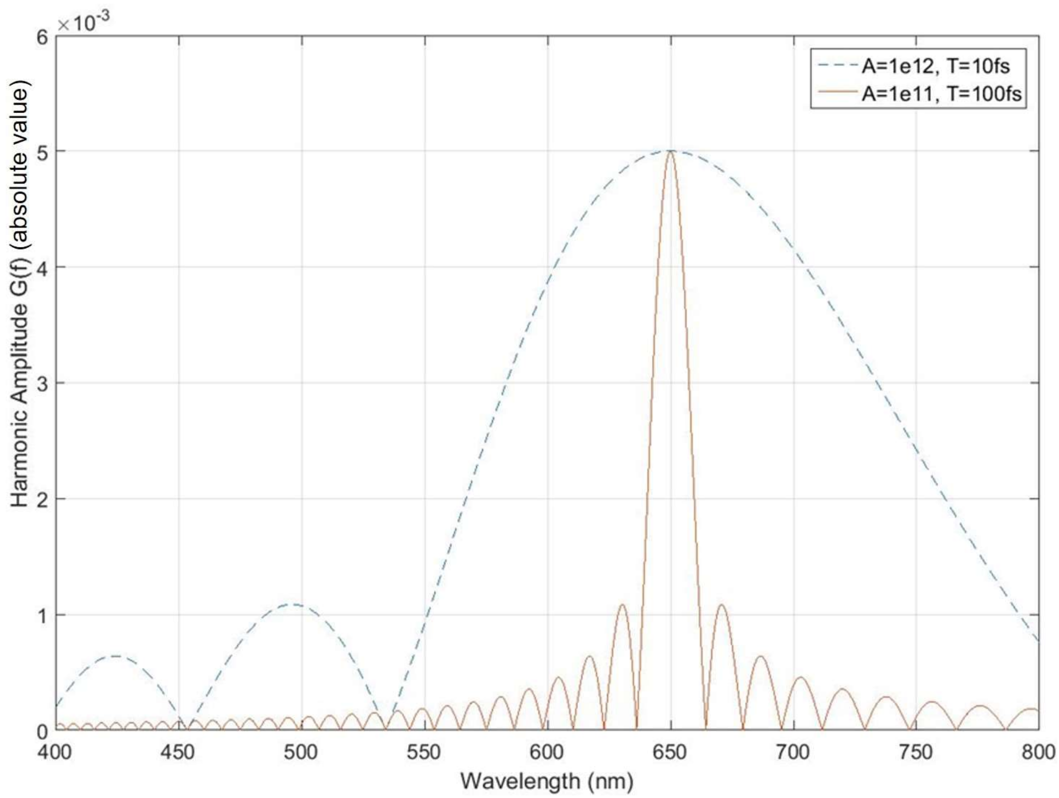

Figure 2.2: Fourier Decompositions of Short-Duration Pulses

[21] Note that actual trillion-volt signals are not assumed for this analysis. Amplitude numbers are chosen for aesthetic depiction in plots.

The solid trace in Figure 2.2 is a *sinc* function[22] representing the harmonic content of a 100 fs pulse of 650 nm light. Especially noteworthy are the non-zero harmonic amplitudes of what this paper calls "primary side-lobes," at 631 and 671 nm. Hence already a problem arises for defining SSR in terms of *pulses* in the highly-dispersive, sub-picosecond domain, because the pulse comprised of 631, 650, and 671 nm harmonics in the frequency domain enters the analysis as a 650 nm energy signal in the time domain.

The problem highlighted by Figure 2.2 materializes when philosophers ask what function an SSR plot like Figure 2.3 is supposed to perform.[23]

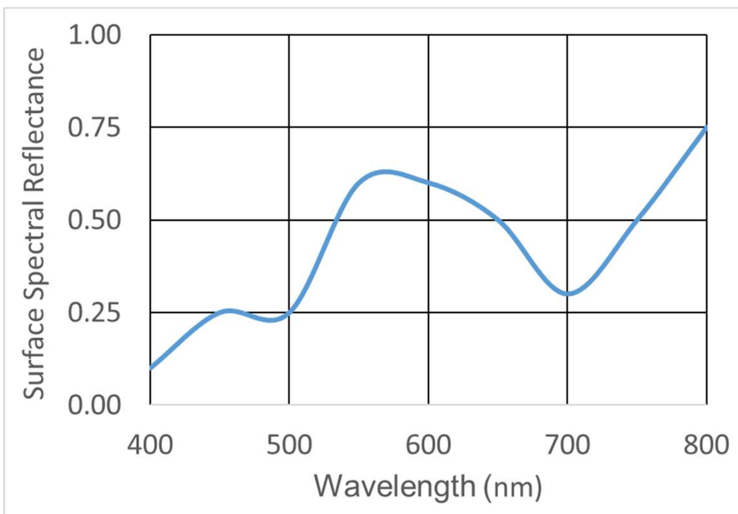

Figure 2.3: Hypothetical SSR Plot

Figure 2.3 is supposed to answer the following question: "How efficiently does this surface reflect 650 nm light?" The canonical response is: "Inspect the SSR plot at 650

[22] Like Figure 2.1 (f), in section 2.

[23] Figure 2.3 is contrived by me, but resembles the SSR plots found in Davies (2014), Kuehni and Hardin (2010), Churchland (2007), Pautz (2006), Byrne and Hilbert (2003), Webster (2002), Hatfield (1992), Hardin (1988), Hilbert (1987), Wandell (1985), and Gaffey (1976).

nm." According to Figure 2.3, the SSR at 650 nm appears to be "50%, on the nose." But Figure 2.2 indicates that when a surface reflects *short-duration* pulses of 650 nm light, the correct response becomes: "Inspect the SSR plot at 631, 650, and 671 nm," which according to Figure 2.3 includes SSR values appreciably above and below 50%.

The upshot is that in the sub-picosecond domain, pulse-SSR (Hilbertian SSR) values are undefined, even operationally. This undefinedness reveals itself by example. Assuming a 5 watt electromagnetic pulse generator and a perfectly reflective surface (SSR = 1), the dotted line in Figure 2.2 indicates that a 10 fs pulse of 650 nm light does not reflect with 5 watts at 650 nm; a substantial portion of the reflected power instead occupies the 496 nm band. For simplicity, let us assume that 20% of the pulse power reflects at 496 nm, and 80% reflects at 650 nm. Now the question is: is the 4 W (80% of 5 W), 650 nm component of the original (or reflected) pulse *itself a 10 fs pulse*? I answer no, lest that 10 fs *component* be interpreted to disperse *again*, *ad infinitum*. There is no fact of the matter, in other words, what the SSR value of the perfect mirror is at 650 nm, in the sub-picosecond domain; for is the power reflected at 650 nm equal to 4 W, or 80% of 4 W, or 80% of 80% of 4 W, or 80% of 80% of 80% of 4 W, etc.? The SSR value at 650 nm is *undefined* below durations of 1 ps, because it is totally arbitrary to determine where to stop the '80% regress' (see previous sentence) for the numerator and denominator of the SSR ratio. Thus *there is no Hilbertian SSR value at the surface* for durations below 1 ps,[24] because there is no such thing as a sub-picosecond *pulse* that reflects at the ostensive 'wavelength' in question (650 nm). To ascribe pulse-reflectance

[24] Recall that this duration threshold changes with pulse carrier frequency (see footnote 11).

to a surface in the sub-picosecond domain is by analogy to ascribe the shape of '*circular-squareness*' to that surface. The ascription is metaphysically void. Whatever reflectance disposition obtains at the mirror, it is not the disposition to reflect *pulses* of light. It is not Hilbertian SSR.[25]

Nor does it help to reply that laser scientists obviously *detect* a reflected, 10 fs, 650 nm component from the perfect mirror, and that this detected 'pulse' has some measured average power. Such a reply begs the question, and equivocates on the meaning of 'pulse'. For laser scientists know better than anyone, perhaps, that *there is no such thing as a monochromatic, sub-picosecond pulse* in the THz range (Hirlimann 2005; Stingl et al. 1995). The option remains open, and indeed appears motivated by the preceding argument, that the disposition manifesting at the perfect mirror is the disposition to reflect the *harmonics* of pulses. If the laser scientist detects a 4 W reflection at 650 nm, she can interpret the Fourier analysis to mean that she is measuring a 4 W, 650 nm harmonic. No, she cannot measure the infinitude of an infinite-duration sinusoid, but she can plausibly measure that 10 fs portion of the harmonic that is not cancelled-out in superposition by its neighboring harmonics (see Figure 2.1 (e)-(f)).

My answer amounts to a straightforward reading of Fourier analysis and intrinsic dispositions. Unlike Hilbertian SSR, the disposition to reflect pulse *harmonics* per-wavelength *can* be intrinsic to surfaces, because that reflectance obtains *despite* harmonic dispersion. Thus for the non-color sciences, I recommend defining 'reflectance'

[25] Because all finite-duration signals disperse, even long-duration signals could be said to suffer a '99.99% regress' akin to the '80% regress' above. This '99.99% regress' is also vicious, and so pulse-SSR is metaphysically undefined for pulses of any duration. I tender this claim in section 4.

(assuming it must be dispositional and intrinsic) as the disposition to reflect the harmonics of pulses. Hilbert's (1987) popular definition in terms of pulses or energy signals should be abandoned. I consider objections in the next section.

4. Objections

I anticipate that a number of objectors may feel the need to defend pulse-SSR (energy-signal SSR or Hilbertian SSR) as a theory of *color* or *color perception*. I would find such objections misplaced. Hilbert and other color theorists already know, for example, that the loss of SSR resolution suggested by my dispersion exercise of section 3 is *miniscule* compared to the loss of SSR resolution to be expected from the human visual system itself (Hilbert 1987, 2281-2283 references the vision model of Maloney and Wandell 1986). I concede this point, and offer no claims against SSR color objectivism (Hilbert's reduction), except the intriguing insight that technically speaking, the SSR dispositions used in color objectivism are not surface properties, but surface-and-medium properties.[26] For pulse-SSR is operationally defined only when impinging light is of sufficiently long duration; I illustrated this limitation with the '80% regress' of section 3. Yet pulse-SSR also fails to be *metaphysically* well-defined, for *all* pulse durations, because the harmonic dispersion generating the '80% regress' generates an equally vicious '99.99% regress' for practically non-dispersive, long-duration pulses. Hence pulse-SSR does not obtain at surfaces *entirely independently* of impinging media, or intrinsically to a surface.

[26] Some color theorists, such as Pasnau (2009), seek to define color as some more-than-surfacial property. It would be premature, however, to suggest that my thesis supports or challenges any such version of color theory.

Some might nevertheless object that the disposition to reflect hyper-picosecond pulses remains intrinsic to surfaces in *color theory*, because in the context of color theory, sub-picosecond events "going on outside of" the surface (McKitrick 2003, 158; see the introduction of this paper) do not matter to color property ascription.[27] Byrne and Hilbert (2003), Hilbert (1987), and Hardin (1988, Chap. 2) all clearly explicate, for example, that correlations of human perceptual reports about color to SSR values are measured only under long-duration, stable, standard illuminants.[28] So the question is how serious Hilbert (1987) is about rejecting the WCC. To the extent that the 'goings on' which differentiate intrinsicality from extrinsicality (McKitrick 2003) are vague or context-specific (e.g. time-scale dependent), then perhaps colors defined as pulse-SSR are 'intrinsic' to surfaces. My point stands, however, that pulse-reflectance *is not* and *cannot be* the 'reflectance' (intrinsic or extrinsic) ascribed to mirrors in sub-picosecond laser science, and I say more about this problem below when I answer a related objection about the optical property known as productance.

In the meantime, some may introduce trouble for my thesis by analogizing reflectance to temperature. Peter Smith (1998) argues, for example, that "*there is no fact of the matter*" (39) about the precise value of temperature defined as the mean kinetic energy (MKE) of substrate molecules, past a few decimal places (40-41). Why? Because that definition relies on molecular velocity, which is determined by modeling molecular motion through a hypothetical sphere, and while the sphere can shrink to a point

[27] I thank an anonymous reviewer for this objection.

[28] Scase and Foster (1988, 196) report a loss of hue discrimination in humans, for example, when pulses "around 500 nm" in wavelength fall from 1 second duration to 3 ms duration (pulse energy controlled).

(precisifying the meters-per-second), velocity is not "perfectly determinate" at a point (40). The Hilbertians who define SSR as pulse-reflectance could use this argument in their defense (since I claimed in section 3 that there was "no fact of the matter" about the value of pulse-SSR), except for some strong disanalogies between temperature and reflectance. Firstly, temperature is not an intrinsic property of objects or surfaces, as SSR is supposed to be. Surfaces are supposed to keep their SSR dispositions under any or no illuminant, but surfaces do not keep their temperatures under any ambient medium, except a constant one. My no-fact-of-the-matter argument about pulse-SSR (section 3) also yields more severe metaphysical implications than Smith's (1998) regarding temperature. The problem of specifying exact temperature is one of narrowing, asymptotic *accuracy* (many decimal places to the right), whereas the putative reflective efficiency of a perfect mirror at 650 nm varies widely and radically, dropping from 100% to 41% in just four iterations of the '80% regress' (80%*80%*80%*80% . . . etc.), thus rendering the pulse-SSR value of the mirror undefined both operationally and metaphysically.[29] Hence compared to the '80% regress' facing pulse-SSR, Smith's (1998) metaphysical problem with temperature looks like a nice problem to have.

Perhaps a yet more cogent objection is the one suggested to me by Christopher Pincock.** Just as the dispositional fragility of a vase needs time to manifest—i.e. the first few nanoseconds of applied force might not stimulate the vase to break or even reveal it to be fragile—so pulse-SSR needs time to manifest its reflective efficiency. I

[29] The '99.99% regress' decreases the alleged SSR value over many iterations rather than a few iterations, but the vicious reduction of a high SSR value to a low one transpires nonetheless.

**During the Q & A of my presentation at *Models and Simulations 8*, Columbia, SC, March 2018.

reply that the time-to-manifest threshold between laser physics and color science proves ambiguous. The pulse duration which the color theorist *needs to exceed* (to avoid dispersion) is the duration that the laser scientist *needs to avoid exceeding* to perform useful work, and yet both parties use the same kinds of mirrors,[30] and call those mirrors 'reflective' objects (Stingl et al. 1995, 603, Fig. 2). There seems to be no indication that various sciences classify different reflectances by their "*attack patterns*," or temporal ramp-up profiles, in the way that sounds or timbres are sometimes classified (Isaac 2018; I quote Kulvicki 2008, 7).

Hence the objection anticipated above,[31] concerning Byrne and Hilbert's (2003) definition of "*productance*," the disposition of a surface "to produce (i.e., reflect or emit or transmit) a specific proportion of incident light" (11), such as on the surface of a translucent "stoplight . . ." (12). The putative challenge to my thesis is that productance is "*relative to an illuminant*," in Byrne and Hilbert's (2003) account, while still being an intrinsic, surface disposition. This distinction may be seen by noting that the formula for productance is "$(ri_{\mathrm{x}} + e)/i_{\mathrm{x}}$,"[32] where r represents the SSR value at some wavelength λ, e is the intensity of λ-wavelength light emitted, and i_{x} is the intensity of impinging radiation of wavelength λ (from illuminant source I_{x}) (12). Thus if i_2 is greater than i_1, then the productance of the surface relative to I_2 is less than the productance of the surface relative to I_1 (at the same wavelength λ). And yet, claim Byrne and Hilbert

[30] In laser design, special "chirped multilayer mirrors" may be needed to condition the duration and bandwidth of a sub-picosecond pulse (Stingl et al. 1995, 602). This exception does not entail, however, that "0.5" reflective efficiency means something different on a chirped vs. non-chirped mirror. There is no difference in meaning.

[31] A version of this objection was raised to me by an audience member at the Graduate Student Conference on Metaphysics, University of Buffalo, October 2018.

[32] Notation changed by me, namely the introduction of "x."

(2003): “Although productances are *relative to* illuminants, it is important to stress that the productance of a surface is *illumination-independent* – that is, independent of the actual illuminant” (12).

The question for my thesis, then, is: at a given wavelength λ, can a surface possess an illumination-independent disposition to reflect sub-picosecond pulses with a different (*viz*. non-existent) efficiency than that at which it reflects hyper-picosecond pulses? More precisely: can *one* intrinsic reflectance disposition possess two illuminant-relative efficiencies, one of which is an alleged efficiency for reflecting sub-picosecond pulses?[33] My answer is an emphatic *no*, for the simple reason that sub-picosecond pulse reflectance is undefined (section 3). I am unprepared to grant that an intrinsic property can be undefined, because I do not know what that claim would even mean. Yet further support for my conclusion hails from the discussion of laser science that transpired two paragraphs ago. Laser scientists and color scientists do not use different, time-scaled ‘types’ of reflective mirror (*qua* reflective), or give any indication that a single mirror possesses *two*, *distinct* reflective efficiencies at the *same wavelength*. A 0.5 reflective efficiency is 0.5, full stop. Reflectance is not “both 0.5 and undefined” at 650 nm. But the pulse-SSR theorist needs to claim that the pulse-SSR value at the surface of a mirror is “both 0.5 and undefined” at 650 nm (see section 3).

Nor does my appeal to scientific practice in the preceding paragraph indicate a merely terminological dispute.[34] My account remains neutral on how many intrinsic or extrinsic dispositions can obtain at a surface, or how many of each kind of disposition the

[33] I thank an anonymous reviewer for pressing me to clarify this question.
[34] I thank an anonymous reviewer for this objection.

sciences should agree exist. I claim only that sub-picosecond pulse reflectance is both operationally and metaphysically undefined, and so cannot be intelligibly ascribed as intrinsic *or* extrinsic to any surface. I secondly claim that pulse-reflectance is metaphysically undefined for all pulse durations, and so pulse-reflectance (Hilbertian SSR) is not intrinsic to any surface. Neither of these problems is resolved by exchanging 'intrinsic' for 'extrinsic' vocabulary. I propose instead to mathematically redefine SSR from pulse-reflectance to harmonic-reflectance, since doing so renders intrinsic per-wavelength dispositional reflectance well-defined (or at least blocks the 80% and 99.99% regresses).

Especially telling in favor of my thesis is how Byrne and Hilbert (2003) construe the productance example. They *define productance* in terms of "monochromatic light of wavelength λ . . ." (12). The consequences of their overlooking the energy-signal versus power-signal distinction in physical optics (section 1) could not be more salient or precipitous. Byrne and Hilbert (2003) only secure productance as an 'illumination independent' yet 'illuminant-relative' surface disposition because they are operating *exclusively and by definition* in the temporal domain of 'monochromatic' light: the hyper-picosecond domain. The productance example succumbs to my thesis instead of answering it, because harmonic dispersion will render the pulse-SSR component of productance (variable *r*, introduced three paragraphs ago) operationally undefined, and so productance itself operationally undefined, in the sub-picosecond domain (and metaphysically undefined for all durations). Metaphysicians of SSR in non-color sciences need to avoid this conflation of energy-signals and power-signals.

In this vein, perhaps some will think that I am abusing a mathematical idealization to infer unsupportable claims about the world, i.e., that harmonics reflect from surfaces. Pincock (2014) assumes without reifying an infinite depth to the oceans, for example, to simplify an equation for water wave dispersion, and Batterman (2010) claims that the taking-of-a-limit in mathematics explains certain regularities about rainbows, without reifying the limited entity (zero-wavelength rays) involved (8). *Contra* Pincock, I am not positing infinities to simplify a complex system or to generate a tractable equation for empirical phenomena. The world has already been described *to me* (by Hilbert and company) as containing a surface disposition to reflect *pulses* of light, and by taking that description seriously, along with empirical and mathematical insights about harmonic dispersion, I find Hilbert's metaphysic of SSR insufficient: harmonic reflectance can be a surface disposition, but pulse reflectance cannot be. Pincock (2014) needs infinitudes *to describe* water waves efficiently (as I might similarly predict harmonic dispersion), but he can return to the finite-ocean world because he does not obviously need to wrestle with tricky (but intuitive) dispositional constraints like the WCC. For is water wave dispersion a dispositional property? Is it 'intrinsic' to a body of water, totally independent of ambient air velocity? To impose or even consider such constraints for a descriptive project about water seems beside the point. In contrast, I posit harmonics to understand what it would take for SSR to obtain as an exclusively *surfacial* (intrinsic) disposition, and so I cannot as easily as Pincock relinquish my 'idealizations'.

Pace Batterman (2010, 8), moreover, I am not taking pulse duration or spectra to any absolute limit in nature, such as wavelength $\lambda = 0$. I am only pointing out a pragmatic temporal lower-bound beneath which humans can no more intelligibly speak of *pulse* reflectance, but beneath which laser scientists nevertheless continue profitable work with what science and common sense treat as a univocal 'reflectance' property above the lower-bound. This univocity is preserved by harmonic reflectance rather than by pulse reflectance. In addition, Batterman's rainbow example (among others) concerns *inter-theoretic reduction* (Batterman 2010, 7), whereas mine does not. I argue entirely from the single science of classical, Maxwellian electromagnetics.[35]

5. Conclusion

I have argued that dispositional reflectance, defined in terms of energy signals or 'pulses' (see sections 1 and 2) cannot be a surface property; pulse-reflectance is only

[35] An anonymous reviewer asks why I define reflectance according to a Fourier basis of infinite-duration harmonics, when e.g. the wavelet transform (WT) correlates finite-duration pulses to *finite-duration* wavelet bases, and so yields fewer "non-zero" coefficients in its representation of a pulse than Fourier analysis yields. I acknowledge that Fourier analysis gives a more dramatic depiction of harmonic dispersion than the WT gives (cf. Deng et al. 2005), and that the existence of a WT option undermines the very notion that optical pulses should be *physically* construed as Fourier superpositions. Nevertheless, I propose moving from the Hilbertian to the harmonic definition of SSR because the latter best respects the 'per-wavelength' intuition so strongly inherent to Hilbertian SSR (see the productance argument of two paragraphs ago in the main text). Some wavelet bases, for example, are non-monochromatic on inspection, and so defining 'per wavelength' intrinsic reflectance according to them is going to be more roundabout (if feasible at all) than is simply ascribing harmonic-SSR to the surface (for a similar problem about the wavelets suitable for audio reconstruction, see Mallat 1999, 546-547).

As the same anonymous reviewer emphasizes, however, many outstanding problems remain for the intimation that pulses could be composed of real harmonics. Palmieri (2012, 533ff) objects that the human ear cannot plausibly process the high number of sinusoids in a Fourier decomposition, and Weatherall (2014) remarks that "in curved spacetime [. . .] Fourier transforms are not generally well-defined" (118, n. 44). Referencing the same source, my anonymous reviewer mentions the existing controversy "that the 'phase velocity' of monochromatic plane-waves can exceed the speed of light in vacuum in materials with an index of refraction less than 1." I grant these points, and so declare harmonic-SSR a suitable starting-point in the metaphysical venture away from pulse-SSR, and not a *unique* solution for defining intrinsic reflectance, and so not an argument for (Fourier harmonic) mathematical realism, or mereological realism about superimposed harmonics.

definable as a surface-and-medium property or extrinsic disposition to reflect pulses whose duration exceeds a (carrier-frequency and application dependent) minimum threshold. What *can* be a wholly surfacial property is *harmonic* reflectance (section 3). The propagating harmonic (in superposition or not), due to its infinite duration, never disperses its bandwidth (see Figure 2.1 (a)-(b)), and so harmonic reflectance—but not pulse-reflectance—can be an intrinsic disposition to respond with a constant per-wavelength efficiency to impinging stimuli.

I interpret such harmonic reflectance to be what Byrne and Hilbert (2003) inadvertently *imply to obtain* at surfaces, for example when they define productance in terms of "monochromatic light . . ." (12). My paper reveals, however, that it is the *harmonic constituents* of a pulse which are monochromatic for any pulse duration, and so harmonic reflectance is (or can be) the per-wavelength disposition that SSR objectivists purport to ascribe as intrinsic to surfaces. Perhaps SSR objectivists (reflectance physicalists) prefer to contextualize the meaning of 'intrinsic' as relative to the hyper-picosecond perceptual capacities of human observers, but I have found it unclear how to square that account with Hilbert (1987) and Byrne and Hilbert (2003), both of which appear in some places to explicitly preclude such a relativization.

In the meantime, metaphysicians of the non-color sciences cannot both call SSR an intrinsic disposition, and define that disposition in terms of pulses (energy signals). Laser science, in particular, should adopt a new, harmonic definition of SSR or reflectance, to the extent that laser scientists (or philosophers) think that all mirrors possess a univocal, intrinsic reflectance property. Indeed, laser science should adopt a harmonic redefinition of reflectance even if scientists and philosophers *deny* that reflectance is univocal or intrinsic, because sub-picosecond pulse-reflectance is not only metaphysically undefined, but even operationally so.

CHAPTER 3

A DILEMMA FOR SEMIREALISM: THE CASE OF FRESNEL-MAXWELL REFRACTION*

1. Introduction

A touchstone for Anjan Chakravartty's (1998; 2007) "semirealist" version of scientific realism, and for other realist accounts (Kitcher 1993; Psillos 1999; Saatsi 2005), is the porting of refraction equations from Fresnelian optics to Maxwellian electrodynamics, a case study popularized in the philosophical literature by John Worrall (1989). Chakravartty's account of refraction deserves scrutiny, however, for a metaphysical difficulty that I find latent within it. This difficulty takes the form of a dilemma, in which the intrinsic dispositional properties by which the semirealist picks out real entities and structure are either not "intrinsic" to their bearers,[1] or not "dispositions" in the traditional sense. By "traditional sense," I mean that as salt always *could* dissolve in the right quantity of water, salt is not always dissolving; to the contrary, I argue that some of the dispositions that Chakravartty posits in the Fresnel-Maxwell case, in order to be intrinsic to their bearers, must be *always* manifesting. And not just in the way that fundamental properties like electric charge might be said to be always manifesting;[2] the

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*Submitted to *Journal for General Philosophy of Science* on February 17, 2021.

[1] "Intrinsic" meaning "not . . . [had] in virtue of standing in a relation to something else . . ." (Chakravartty 2013, 43).

[2] French (2013, §2) and Chakravartty (2013, §2.4) discuss this example for semirealism (which I revisit in sections 3.3.1 and 3.4.1), but the example lacks the structural implications afforded by other property ascriptions, which I explain in due course.

"always" that I identify means that the manifestation occupies an infinitely-sized real structure in the temporal dimension.[3]

While I think that a mild revisionism about dispositionality could allow the semirealist to retain intrinsic dispositions about refraction (section 3.3.1), I will argue that even these repaired dispositions manifest real structure that is eternal in its temporal extent.[4] This implication of infinitely-sized structure, while not in my view a *reductio* of semirealism, is to my knowledge unprecedented in realist interpretations of the Fresnel-Maxwell case, and so deserves to be amplified. My central argument is that semirealism's commitment to the "intrinsicality" of some dispositions is what entails their infinite structure and consequent trouble for the "disposition" concept. Hence the semirealist dilemma will arise in any account that ascribes electromagnetic properties as intrinsic dispositions (e.g., Byrne and Hilbert 2003; Jackson 1998, 87; Ellis 1992; Hilbert 1987).

My first step in motivating the semirealist dilemma is twofold. In the next section, I summarize the apparatus of properties and structural relations that semirealism employs. I also transition away from the usual terminology of the Fresnel-Maxwell case study. Whereas semirealists and others discuss how Fresnel's laws relate the *intensity of light* to light's angle of refraction, I transition to the equivalent discussion of how Fresnel's laws relate the *refractivity of a medium* (i.e., its refractive index) to light's angle

[3] The equation for the electric field amplitude of a point charge possesses no temporal variable (Halliday et al. 1997, 557), and so I deny that always-manifesting charge requires infinitely-sized real structure in the temporal dimension. This denial respects Chakravartty's "minimal demand" heuristic that I explain in section 2.

[4] By the same argument, these dispositions also require infinitely-durative stimuli, but I will mostly ignore this implication, to avoid cluttering the discussion.

of refraction. I say "equivalent," because as I will explain in section 2, the intensity of light is definable in terms of the refractivity or refractive index of its medium of propagation.

For the time being, I ask the reader to accept that this translation from intensity to refractivity, undertaken for expository convenience, is philosophically neutral. I could run my argument in terms of light's intensity, and so I make no unfair claims against Chakravartty, who to my knowledge discusses only intensity and never mentions "refractivity" in his writings. Nothing about my argument depends, furthermore, on whether Fresnel and his contemporaries recognized the interdefinability of intensity and refractivity, or whether dispositional realists *should* focus on the intrinsic properties of light versus media, etc. I translate from intensity-talk to refractivity-talk because the *intrinsic* difference between mica and quartz, for example, seems easier to conceptualize than the intrinsic difference between two differentially intense light pulses, especially once I introduce the "per wavelength" dimension of intensity and refractivity (in section 3.1).

Another advantage of my translation is the narrative continuity achieved by comparing semirealism with antecedent realist accounts of other dispositional properties of electromagnetic media: namely David R. Hilbert's (1987) influential account of reflectance (a counterpart of refractivity, described by Fresnel's laws), and Brian Ellis's (1992) account of X-ray diffraction, which I discuss in section 3.4. To remind the reader that everything I say about refractivity applies *mutatis mutandis* to intensity, however, I will sometimes write "(intensity)" parenthetically, or add a footnote. I conclude in section 4.

2. Background on Semirealist Refraction

One attraction of the Fresnel-Maxwell case study, for semirealists and others, is the support that it appears to provide for structural realism. Structural realism, in turn, takes its motivation from the hypothesis that science succeeds progressively through the centuries because it truly refers to *something* in the world, but because that "something" routinely turns out not to be entities—like the luminiferous ether that Fresnel posits for light propagation, but which contemporary science rejects—then the real referents of scientific theory are the mathematically articulated structural laws that port from theory to theory.[5] Finding some forms of structuralism too abstract to be *realisms*, however, Chakravartty (2007, 39 ff.) insists that real structural relations possess real relata (40), and he picks out these relata by specific properties that they possess. These properties he calls "*detection* properties" (47), intrinsic dispositions (62, 120) that inhere in entities, and which are identified by their being "causally linked to the regular behaviours of our detectors" (47). This definitional dependence of detection properties on the instrumentation employed across scientific epochs is essential, Chakravartty (2007) argues, to the predictive success of the laws that relate those properties and survive theory change (50). He calls theoretical properties not so related to instrumentation "auxiliary properties" (Chakravartty 1998, 395), and on their ontology he is "agnostic" (Chakravartty 1998, 402), pending any empirical tests that might confirm them (Chakravrtty 1998, 404).

[5] Wright (2017) demurs that the refraction case study is a poor one for structural realism, since Maxwell did not relinquish belief in an ether. I ignore this historical detail to analyze the metaphysics of semirealism, my conclusions about which apply even if the Fresnel-Maxwell case study is sub-optimal for defending realism.

That detection properties are "intrinsic" to their bearers, Chakravartty (1998) seems to hold by the straightforward inference that the best candidates for real entities are those whose properties facilitate detection by *various* instruments or measurement procedures (394, 397), such variety suggesting that the entity in question possesses its properties in the absence of measurement, or intrinsically. Otherwise, Chakravartty (2007) simply postulates dispositional realism as a tenet of semirealism (42); dispositions manifest when stimulated, and the manifestations of detection properties he calls "concrete structures" (150) and "causal laws" (116), such as Fresnel's laws of refraction. While I will not criticize any of these semirealist tenets in this paper, I will argue that positing refractivity as an intrinsic disposition of media (like air, quartz, the free space vacuum, or mica slabs) commits the semirealist to infinitely-sized structure in the temporal dimension. As I mentioned in section 1, this implication is both foreign to most realist discussions of the Fresnel-Maxwell case, and it problematizes the concept of dispositionality employed by the semirealist (section 3).

To motivate the semirealist to ascribe refractivity at all, however (since Chakravartty never mentions it; see section 1), I simply employ the semirealist's own method for ascribing detection properties. That method, as described by Chakravartty (1998, 396) is to "turn to the equations" like Fresnel's laws that survive theory change, "and ask: what do these mathematical relations *minimally demand*" ontologically? The minimal demand is about necessity, not possibility (396): which detection properties would we *need* to posit to make sense of the detections that Fresnel performed and from which he constructed his laws? The idea is that considering the minimal demand might help us distinguish light's intensity from intensity-in-an-ether, ether being an auxiliary property, just as absolute space is auxiliary to and not minimally demanded by Newton's laws of motion (Chakravartty 2007, 53).

I begin my "minimal demand" analysis of Fresnel's laws, then, by examining one of them (quoted from Chakravartty 2007, 49):

$$X'/I' = 2 \sin r . \cos i / \sin(i + r) \qquad [1]$$

Variables r and i, in Equation 1, represent the refracted and incident angles of light measured normal to a refracting surface, and X' and I' the intensities (squared amplitudes) of the polarization vectors, orthogonal to the plane of incidence, for the refracted and incident rays, respectively.[6] The sine and cosine functions in Equation 1 are trigonometric identities, and so do not represent wave oscillation, but only vector directions. Thus, we have in Equation 1 one fourth of a sketch of how light refracts through the interface of any two (unspecified) media.

Chakravartty (2007, 49) takes Equation 1 to minimally demand two detection properties: the "intensities" and "directions" of light's propagation.[7] If light and other electromagnetic radiation possess these properties, then they explain why Fresnel and Maxwell observe radiation to obey Fresnel's laws in different epochs.[8] I can equivalently

[6] Additional equations not shown describe the intensity and angle of *reflected* light, a standard by-product or counterpart of refraction.

[7] Important to note is that the earlier Chakravartty (1998) lists the Fresnel-Maxwell detection properties of light with the disjunction "amplitudes or intensities," and the disjunction, "angles or directions of propagation" (396). I flag this detail because I appeal explicitly to amplitudes in my argument of the main text below.

[8] I mention here but leave aside the intriguing objection by Saatsi (2005, 525), that intensity was not a plausible detection property *for Fresnel*, because Fresnel assumed

commit the semirealist, however, to positing refractivity—i.e. the index of refraction—of *either* the refracting medium or the medium of incidence, as a detection property related by Fresnel's laws to the refraction or incidence angles, because the intensity of light and the refractivity of its medium are intertranslatable. This point is complex, but it is worth going over, because it eventually streamlines this paper's philosophical discussion: my claim is that intensity X' in Equation 1 is redefinable in terms of the refractivity of the "refracting medium"[9] (mica, quartz, etc.), and that intensity I' in Equation 1 is redefinable in terms of the refractivity of the medium of incidence (usually ambient air in the laboratory). I am not claiming that this translation is especially useful for science, or anticipated by Fresnel and Maxwell, or improves any argument for realism. I am simply pointing out that when the semirealist references the "intensity" of light in a medium, she implicitly references the "refractivity" of that medium; the morning star is the evening star, if you will, for light only possesses intensity because it propagates in a medium of specific refractivity (see below). Of course, it is important not to accidentally swap mediums in speech, thought, or writing: the intensity of light in air is not the same property as the refractivity of quartz; that monistic blurring of property identities would be indefensibly strong at present, and I will not revisit it in this paper. But as long as media reference-swapping is avoided, refractivity occupies the same location as intensity (with qualifications, see below) in the concrete structure of Fresnel's laws.

more than measured light's transverse oscillatory character, from the domain of mechanics.

[9] I use scare quotes because as I will explain, all media besides the free-space vacuum refract radiation along a per-wavelength dimension (Halliday et al. 1997, 857).

Here is one way to understand the interdefinability of intensity and refractivity. The intensity of light is definable in terms of the index of refraction ("refractivity") of the medium through which that light propagates, because both intensity and index of refraction are definable in terms of the medium's electrical permittivity ε (in $N*m^2/C^2$, where C is Coulombs of charge) and magnetic permeability μ (in Tesla-meters per Ampere). A number of complex factors determine the ε and μ of a given medium, including its chemical composition (Cheng 1992, 342). Nevertheless, the intensity of light is definable in terms of ε and μ, because intensity is amplitude-squared, said amplitudes being the oscillatory electric and magnetic fields of propagating light. Whereas electric field amplitudes are (generally speaking) inversely proportional to ε (Halliday et al. 1997, 557), magnetic field amplitudes are (generally speaking) directly proportional to μ (Halliday et al. 1997, 729). Hence intensity can be expressed in terms of ε and μ.

A medium's index of refraction n, on the other hand, is colloquially and correctly defined as the speed of light in vacuum (c) divided by the speed of light in the refracting medium (Halliday et al. 1997, 903); but the speed of light in any medium is a function of the ε and μ of that medium.[10] Hence Fresnel's laws are commonly expressed, in discussions simpler than Chakravartty's (2007; 1998) and Worrall's (1989), as (citing Halliday et al. 1997, 856):

[10] This dependence partially explains the frame-invariance of the speed of light in a given medium. The velocity of a light beam shining from a 50 kph locomotive, for example, toward a detector placed in the direction of locomotion, will *not* be measured as 50 kph greater than the light from a stationary candle next to the tracks, because light propagates not with an additive velocity but according to μ and ε of the medium.

$$n_2 \sin \theta_2 = n_1 \sin \theta_1 \qquad [2]$$

where n_1 is the index of refraction (i.e. "refractivity") of the medium of incidence, n_2 the index of refraction of the refracting medium, θ_1 the incident angle of propagation measured from the normal to the media interface, and θ_2 the refracted angle of propagation measured from the normal to the media interface. Although not perspicuous within Equation 2, the unitless index of refraction is a function of ε and μ just as intensity is, and so intensity and refractivity can be written in terms of each other, as I illustrate in the following proof-sketch:

1) The intensity of light in air is the square of electric field E, or magnetic field B.[11]

2) The *speed* of light in air is E/B (Halliday et al. 1997, 844, 846-847).

3) The refractivity (index of refraction) of air is $n_a = c/(E/B)$.

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4) E^2 (intensity) $= (cB/n_a)^2$

According to step (4) above, the intensity of light in a given medium is a function of the refractivity (n) of that medium, and vice-versa, so the semirealist ascribing dispositional intensity is at least *prima facie* committed to the reality of dispositional refractivity as a detection property; the two properties are intersubstitutable (with appropriate scaling and inversion) in the concrete structure of Fresnel's laws.[12]

[11] Depending on polarization, a detail I can omit for clarity.

[12] The semirealist might reject this commitment to intrinsic refractivity by citing Wright (2017), who informs us that according to contemporary science, "the refractive index of bulk glass or quartz is seen as an average effect of more fundamental interactions" (46).

3. The Infinite Structure Manifested by Intrinsic Dispositional Refractivity

3.1 The Per-Wavelength Pulse Regress

With the technicalities of section 2 out of the way, I can now explain the dilemma that I perceive for semirealism: namely, that once we ascribe intrinsic dispositional refractivity to media,[13] we launch a conceptual regress about refractivity that is most readily blocked, I will argue, by defining refractivity's manifestation as infinitely durative. Such a bizarre problem and drastic-looking solution may appear unmotivated, or implausible, but as I have argued elsewhere (Danne 2020), they are actually very proximate consequences of ascribing most any electromagnetic property as real. The problem is that electromagnetic properties are only *usefully* ascribed along a "per wavelength" dimension. The prism splits the white light into a rainbow, for example, because the 700 nm red component and 400 nm blue component propagate at different speeds. The refractivity of a medium is wavelength-dependent (Halliday et al. 1997, 857), and so are the reflectance of a surface, the diffraction pattern of X-rays, and even the intensity of light (as I will implicitly argue).

The trouble with ascribing per-wavelength electromagnetic properties, however, is that light[14] never propagates at one wavelength or frequency.[15] Nor should this scientific fact be understood *solely* as the result of a distorting medium, measurement

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But by the argument just given in the main text, Wright's charge falls equally against the intensity (E^2) ascribed by semirealists.

[13] Which the semirealist inevitably does when ascribing intrinsic intensity to light (section 2).

[14] Or other electromagnetic radiation from invisible parts of the spectrum (infrared, radio, radar, X-ray); I assume this qualifier throughout.

[15] As affirmed by the authoritative Born and Wolf (1999): "light produced by a real physical source is never strictly monochromatic . . ." (286). Frequency is the speed of light divided by wavelength.

noise, or a noisy source. The inverse relationship of a signal's duration to its bandwidth, a relationship that I call "harmonic dispersion,"[16] is instead a non-quantum (Hirlimann 2005, 31), widely empirically-confirmed law of classical electromagnetics, and I argue in this section that harmonic dispersion undermines the conceptual coherence of dispositional refractivity (intensity), rendering it non-ascribable as an "intrinsic" property in the ordinarily understood sense.[17] The ordinarily understood sense of intrinsic refractivity, I venture, is the disposition of a medium to refract *finite-duration* pulses of light with the intensities and angles specified by Fresnel's laws. (Indeed, with what other durations of light did Fresnel work, or would any other human scientist work?) This practically-minded definition of refractivity, however—an entirely reasonable one to assume that the semirealist would adopt—will not work for the metaphysics of semirealism.

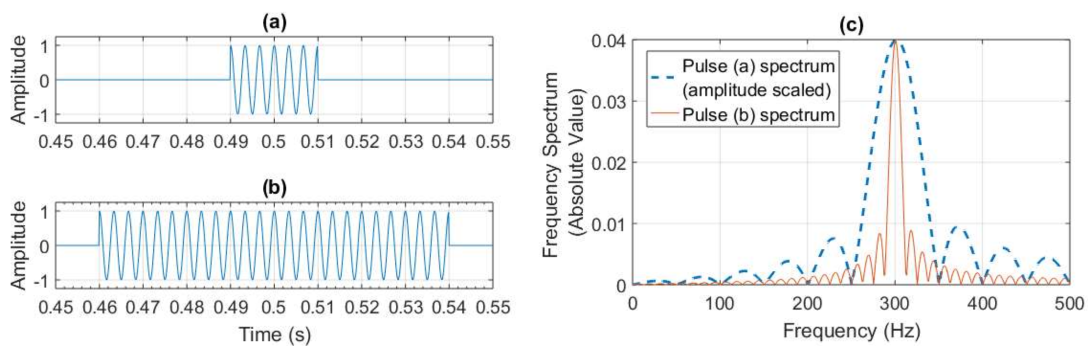

Figure 3.1: Harmonic Dispersion [4x duration]

[16] The similarly named "chromatic dispersion" is the spreading of the rainbow through the prism, due to refraction. Harmonic dispersion is something different, as I explain in the main text.

[17] More carefully: harmonic dispersion *compounds* any signal bandwidth expansions due to noisy sources, etc., but these sources of noise by themselves do not generate the conceptual regress that harmonic dispersion generates.

To see the metaphysical problem engendered by harmonic dispersion, let us first examine it. Figure 3.1 reveals the inverse relationship of a pulse's duration to its bandwidth, since pulse (a) differs from pulse (b) only in its shorter duration, but pulse (a) exhibits an appreciably higher bandwidth of frequencies (dotted trace, plot (c)) than the longer-duration pulse (b) exhibits (solid trace, plot (c)). The upshot is that finite-duration pulses (a) and (b) do not propagate "per wavelength," or at the 300 Hz depicted in Figure 3.1 (a) and (b), but rather as a wave packet centered at 300 Hz. From this observation, I will argue that ascribing to a medium the per-wavelength disposition to refract finite-duration pulses of light launches a vicious conceptual regress; and regressive properties, I will argue, are implausible candidates for so serious a role as that of *intrinsic* disposition.[18]

The conceptual regress facing per-wavelength electromagnetic property ascription, I call the "per-wavelength pulse regress" (PWP), and I illustrate it now. In the case of refractivity, we should ask in general terms what is happening per-wavelength; yes, light is propagating in a direction, but an electric field of some intensity is also oscillating. This oscillation can be expressed as a per-wavelength instantaneous power, average power, or energy, etc. For continuity with Hilbert (1987, Chap. 3), who defines reflectance in terms of pulse average power, let us also choose average power.

[18] Real light pulses possess center or carrier frequencies in the hundreds of terahertz (THz; 10^{12} Hz), far above the 300 Hz signals that I use in Figure 3.1 for ease of modeling. Nevertheless, harmonic dispersion afflicts all finite-duration pulses of all useable carrier frequencies, and is empirically detected across a wide range of electromagnetic applications, from radar (Haykin 1989, 36-37) to laser physics (Deng et al. 2005; Stingl et al. 1995).

Assume, then, that a monochromatic source generates a 600 nm pulse[19] with average power 5 Watts (W). By Figure 3.1, we know that this pulse will not dissipate *all* of its average power at 600 nm. Indeed, laser science reveals that at sub-picosecond (< 1 ps; 10^{-12} seconds) optical pulse durations, harmonic dispersion can be as wide as 100 nm (Deng et al. 2005; Stingl et al. 1995), which means that appreciable power will dissipate at 550 nm and 650 nm, besides at the 600 nm center wavelength.[20] For easy computation, let us estimate that 80% of the 5W pulse propagates at 600 nm, and 20% propagates at auxiliary wavelengths. (The accuracy of this estimate will not matter, because a regress is a regress.)

Hence, we have a pulse propagating toward a refracting medium, the pulse possessing 4W at 600 nm, and 1W at other wavelengths. The key question is: will this 4W *component* or *portion* of the total pulse *itself* disperse *all* of its average power at 600 nm? Consider what the semirealist should reasonably anticipate. According to Fresnel's law in Equation 2, the light will refract at whatever angle θ_2, appropriate to n_2, for wavelength 600 nm (assuming 0W lost to reflection, for the sake of argument). There is no mention of "wavelength" in Fresnel's laws (Equation 1 or 2), so a straightforward application of Fresnel's laws suggests "Yes," all 4W are going to refract at θ_2. But I say, "No," all 4W are *not* going to refract at θ_2, because harmonic dispersion is a law of nature. The 4W component of the original 5W pulse has *the same duration* as the original 5W pulse (what other duration would the component have?), and *ex hypothesi*,

[19] Analogous to the "300 Hz" pulse shown in Figure 3.1 (a).

[20] Analogously to the harmonic dispersion of Figure 3.1 (c) dotted trace, which dissipates not only at 300 Hz, but also appreciably at ~225 Hz and ~375 Hz sideband peaks.

propagations of *that* duration disperse 20% of their average power to auxiliary wavelengths. So, the 4W component at 600 nm actually dissipates 3.2W at 600 nm (80% of 4W); but wait, that cannot be right, because the 3.2W component is just as short in duration, and so disperses to 2.56W (80% of 3.2W), which actually disperses to 2.05W (another 80%), etc., *ad infinitum*. This diminution of pulse average power to zero, for any given wavelength, is the PWP regress.

Granted, the PWP regress is not a *physical* diminution; the scientist likely measures 4W refracting at 600 nm and thinks nothing further of it. The point, however, is that what the scientist measures is not a finite-duration *pulse*, by the argument just tendered. Fresnel's laws, as written in Equations 1 and 2, *do not apply to pulses*, because pulses do not propagate along the "per wavelength" dimension by which refractivity is specified. This result is counterintuitive for two reasons. Firstly, because finite-duration pulses appear to be all that a scientist could possibly manipulate in any epoch, and so Fresnel's laws *must* apply to them, or apply to nothing. Secondly, we cannot just force the issue and say that Fresnel's laws apply to pulses, because the consequence of that assertion is that a 5W 600 nm pulse refracts with infinitesimal (~0W) average pulse power at 600 nm, a contradiction. If we cannot say that "pulses" are refracting, then we cannot say that pulse-refractivity is intrinsic to media.

We are left with a property (pulse-refractivity) too conceptually incoherent to ascribe as a real property. "Per-wavelength pulse refractivity" is a contradiction in terms, like "non-cubical cube," because "pulses" do not propagate per-wavelength. Even if "non-cubical cubes" could be construed as intrinsic properties,[21] such a construal is

[21] Marshall (2020) discusses some such views.

clearly not how scientists, semirealists and other philosophers, laymen, or Fresnel's laws represent refractivity and other optical properties. To call refractivity "intrinsic" to media,[22] we need to know *what* is refracting per-wavelength, since the answer is not "pulses" or finite-duration propagations of light.[23,24] This conclusion is a significant strike against the applicability of Fresnel's laws, and against the metaphysics of calling them concrete structures, since concrete structures relate intrinsic properties. As I understand semirealism, concrete structures should not, or cannot, obtain between "intrinsic" properties that are self-contradictory, like non-cubical cubes.

3.2 Blocking the Per-Wavelength Pulse Regress

Interestingly, a solution to the per-wavelength pulse regress (PWP) is near at hand. This solution renders dispositional refractivity *n* intrinsic to media, and allows us to identify something besides a "pulse" as that which dissipates power at 600 nm (in the example of section 3.1). This solution finds an early precedent, moreover, in the writings of Fresnel himself (see section 3.3.2). Refractive index or dispositional refractivity can

[22] As I argued in section 2 that the semirealist is committed to doing. Attempting to ascertain the "intensity" of a pulse, rather than its average power, would also launch the PWP regress.

[23] Note that abandoning intrinsicality, and calling refractivity an *extrinsic* disposition between media and light, offers no help here. As long as we discuss the propagation of "per wavelength" pulses, we incur the PWP regress. (For more on extrinsic dispositions, see Hoffmann-Kolss 2010 and McKitrick 2003.) The semirealist might free herself from the "intrinsicality" horn of the dilemma by going operationalist about properties, defining them as a set of measurement instructions for obtaining a phenomenal result, but Chakravartty (2013, §2.1) appears to reject this move.

[24] One might object that I run the PWP regress from the sub-picosecond domain, which is not a domain that Fresnel manipulated; in reply, I appeal to Figure 3.1 and to Haykin (1989, 36-37) to reinforce the fact that the PWP regress also afflicts much longer-duration signals of any useable frequency. Even a 99.99% regress, which iterates slower than the 80% regress that I assumed three paragraphs ago in the main text, plunges pulse average power asymptotically to ~0W.

be intrinsic to media, I submit, if we redefine it from the disposition to refract finite-duration pulses of light, to the disposition to refract the Fourier harmonics that superimpose without remainder into pulses of light. Fourier harmonics are infinite-duration monochromatic sinusoids, and I have already employed them in Figure 3.1. Every point of the solid or dotted traces in Figure 3.1 (c) represents a harmonic of the corresponding amplitude and frequency shown in that graph,[25] and they superimpose without remainder into the finite-duration pulse shown in (b) or (a), respectively.

Harmonic-refractivity can be intrinsic to media, because the infinite duration of the harmonic gives it unity bandwidth; hence the harmonic will never suffer harmonic dispersion or launch the PWP regress. When a scientist measures a 4W "component" refracting at 600 nm in my account (per the example of section 3.1), she is not manipulating a "pulse," even though a finite-duration propagation is all that she observes. What she observes, by my redefinition, is the finite-duration *portion* of a 600 nm *harmonic*, a portion which for that duration is not cancelled-out in superposition with other harmonics. Precisely the same situation holds in Figure 3.1. Pulse (a) has zero amplitude below 0.49 seconds and above 0.51 seconds, *because* its 600 nm harmonic and auxiliary harmonics superimpose with the weights shown in Figure 3.1 (c).[26] In this way,

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[25] Fourier analysis additionally employs negative-frequency harmonics, which I omit for clarity. See Haykin and Van Veen (1999, Chapters 3 and 4).

[26] More precisely, Fourier analysis is time-invariant, so *any* 20 ms pulse of 300 Hz carrier will have the dotted spectrum shown in Figure 3.1 (c); starting the pulse at 0.49 seconds is my random but necessary artifice in constructing the graph. Figure 3.1 also phase-shifts the weighted harmonics according to a per-wavelength phase plot omitted for clarity.

the Fourier interpretation of dispositional refractivity blocks the PWP regress and renders refractivity intrinsic to media.[27]

3.3 Implications and Precedents of Harmonic-Refractivity, for Semirealism

3.3.1 The Dispositional Refractivity (or Semirealist) Dilemma

Should semirealists accept the harmonic-refractivity solution to the PWP regress? Doing so introduces into semirealism an infinite-duration structure, namely that of Fresnel's laws, which strictly speaking apply only to propagations of a single wavelength, which as we have seen from sections 3.1-3.2 means infinite-duration harmonic propagations. While I have found neither prohibition nor endorsement of infinite structure in Chakravartty's writings, accepting it spawns the dilemma for semirealism mentioned in section 1: either refractivity (intensity) is not "intrinsic," because not defined with harmonics, or refractivity (intensity) is not a "disposition," because dispositions do not typically possess the *eternal* stimuli and manifestations that harmonics are. To borrow Chakravartty's (2013, 45) words, the existence of infinite structure threatens to "dissolve dispositions into manifest relations."

In response to this dilemma, perhaps the semirealist can simply accept the reality of eternally-manifesting dispositions. So long as portions of eternal manifestations (like superimposed harmonics) are in-principle undetectable (because their net amplitude is

[27] One might object that I unfairly gravitate to the Fourier representation of waves, when more compact mathematical representations like wavelets are available, the latter of which do not employ a "harmonic" base for composing and decomposing time-domain signals. I reply that the onus is on the objector to show that a representation besides Fourier's blocks the PWP regress (section 3.1). Wavelet bases, for example, are heterochromatic, and therefore suffer harmonic dispersion and launch rather than block the PWP regress (Mallat 1999, 546-547; Deng et al. 2005).

zero[28]), then that disposition still functions as a detection property, because it is only detected when its net manifestation is non-zero.[29] The semirealist might have no metaphysical need of an always-manifesting disposition to *in fact* behave traditionally (like the salt that always *could* dissolve), in other words, if *for all the semirealist knows* the always-manifesting disposition appears to behave traditionally. The semirealist might allow detection properties to be metaphysically always manifesting, if they are epistemically traditionally behaved. Harmonic-refractivity could function as such an intrinsic disposition.[30]

In his reply to Steven French (2013), Chakravartty (2013) considers the problem that relations between "elementary particles . . . are always manifesting" (47). Whereas sodium chloride is not always dissolving in water, its charged atoms can be construed as always manifesting an electric field. Part of Chakravartty's reply to this problem seems pertinent to that of harmonic-refractivity:

> Even if it were true that the relations of fundamental physical entities are always manifesting, one might nevertheless favour the economy of an account of properties and laws that applies across the sciences, not merely to basic physics. The account of properties and laws I elaborate has this virtue. (Chakravartty 2013, 47)

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[28] As in Figure 3.1 (a) below 0.49 seconds and above 0.51 seconds.

[29] As in Figure 3.1 (a) between 0.49 and 0.51 seconds.

[30] Psillos (2013, 37) objects that the semirealist distinction between detection and auxiliary properties is more "pragmatic" than "epistemic," because detection properties are by definition just those which we have *in fact* managed to measure (for reply, see Chakravartty 2013). One might redouble this "pragmatic" objection at a metaphysical level, against my proposal that eternally-manifesting dispositions are dispositions *because* they are occasionally detectable by us (*viz*., when they are in nonzero superposition). The objection would be that dispositionality should not depend on detectability, but vice-versa. I think that this objection returns the ball to the semirealist's court, however, and is not one that I should attempt to resolve in this space.

Just as I found scientific objections to the “intrinsicality” of refractivity unreasonably severe in footnote 12, so Chakravartty by this block quote could construe objections to the eternal manifestation of intrinsic refractivity as unreasonably severe. While the burden remains with the semirealist to decide how appropriate it is to treat harmonic-refractivity as if it were a traditionally-behaved disposition, a wholly separate consideration supports the semirealist adoption of infinite structure for Fresnel-Maxwell refraction: the writings of Fresnel himself. I briefly recount some of those insights now, to show that infinitely-sized structure is by no means unprecedented in Fresnel’s influential theory of light.

3.3.2 Fresnel on the Infinite Structure of Physical Optics

Well-documented by Juha Saatsi (2005) are the mechanical assumptions by which Fresnel developed his refraction laws. Some of these assumptions appear also in Fresnel’s (1900) award-winning memoir on diffraction, which is the interference patterning of light partially occluded by a sharp edge. Relevant to my arguments is Fresnel’s assumption that light accelerates ether particles in a transverse wave motion (102-103), a wave profile resembling in its essentials today’s Fourier harmonic (cf. Cheng 1992, 356, and Worrall 1994, 340). Indeed, Fresnel sometimes explicates and exploits the *infinite* duration of monochromatic (harmonic) light within his theory.

For example, in explaining why the light diffracting through a narrow two-edged slit some distance L from a light source produces rings on a receiving surface that are *brighter*[31] than the glow from light's diffraction across a single, sharp edge,[32] Fresnel (1900) argues that the brightness differential *reverses*[33] when "L is at an infinite distance" from the slit and edge (140). By my literal reading of Fresnel, this deference to infinite L suggests the propagation of Fourier harmonics, since only infinite-duration harmonics could actually effect the brightness reversal from an infinite distance; experimenters would otherwise wait forever for a finite-duration pulse to complete the journey.[34] Less literally, references to infinite L imply that if L is *large enough* relative to the slit width, a brightness reversal can be effected, and an infinite L generalizes this observation. Nevertheless, Fresnel finds reference to infinite structure sufficiently important to commit to print.

Another appeal to the infinite appears in one of Fresnel's footnotes:

> I am here discussing only an infinite train of waves, or the most general vibration of a fluid. It is only in this sense that one can speak of two light waves annulling one another when they are half a wavelength apart. The formulae of interference just given do not apply to the case of a single wave, not to mention the fact that such waves do not occur in nature. (Fresnel 1900, footnote p. 108)

This quote implicitly echoes my argument from sections 3.1-3.2, that finite-duration monochromes "do not occur in nature" (due to harmonic dispersion), but that infinite-

[31] Fresnel (1900) says "the light . . . will be increased . . ." (140), which I interpret to mean "the intensity will be increased." My uncertainty on this technical point does not affect my argument.

[32] See Buchwald (1989), chapters 5 and 6, for accessible summaries of these experiments.

[33] *Viz*., the single edge diffracts more brightly than the slit.

[34] Fresnel (1900) does not propose this "journeying" argument, but I think that it applies. He instead infers from infinite L a geometrical insight about the Huygens model of spherical wave propagation (140). Contemporary scientists still use this geometrical model to explain refraction (e.g., Halliday et al. 1997, 903).

duration waves (an "infinite train") would superimpose cleanly into pulses and so reflect "per wavelength." Said another way, Fresnel all but admits in this quote that his refraction laws (Equations 1 and 2) literally pertain only to infinite-duration propagations.[35] The semirealist, then, would not be unprecedented in explicating an infinitely-sized structure for refraction, and in proportion to her joint commitment to dispositionality and intrinsicality, she should do so.[36]

3.4 Why the Harmonics of Dispositional Refractivity Should Not be Construed as Idealizations

Taking stock, I have argued that intrinsic dispositional refractivity requires harmonic structure for its stimulus and manifestation (sections 3.1-3.2), that this requirement generates a dilemma for semirealism that a mild revisionism about

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[35] Because refraction uses the same Huygens model as the diffraction under discussion.

[36] An additional spate of evidence supports the precedent of infinite structure in Fresnelian optics, but also bogs down my paper's main narrative, so I present that additional evidence here. Kipnis (1984), for example, quotes Fresnel from a source besides the diffraction memoir, as claiming that "the representation of light waves by [the Fourier harmonic] relates only to the regular part of a wave train, which could be considered as *infinitely large* comparatively to its ends" (207, emphasis mine). This quote reinforces the point I made in section 3.1, that introducing "ends," or finite duration to a pulse, blows up its frequency bandwidth (cf. Wilson 2018, Chap. 5, §iii); in Fresnel's work, the methodological equivalent of this blowup ruins superpositions important for the Huygens model.

Other authors point out that the hypothetical diffraction gratings in Fresnel's theory require infinite length in the direction(s) away from the edge or slit, to effectively block all grating-side secondary waves in the Huygens model (Cantor 1983, 153; Buchwald 1989, 182). The very contest that declared Fresnel's diffraction memoir the winner, moreover, instructed contestants "[t]o determine by precise experiments all effects of diffraction of direct and reflected light rays when they pass separately or simultaneously near the extremities of one or several bodies of *limited or indefinite extent*, taking into account the intervals between these bodies, as well as the distance from the luminous body from which the rays emanate" (Kipnis 1984, 244, emphasis mine). I read "indefinite" as the opposite of "limited" in this passage, namely as "unlimited" or infinite (hence the infinitely-long diffraction gratings, or "bodies"). These quotes suggest that infinitely-sized structure is a key component of Fresnelian optics.

dispositions could resolve (section 3.3.1), and that infinitely-sized structure anyway finds a clear precedent in Fresnel's work (section 3.3.2). One problem remaining, however, is that semirealism is supposed to be a *realist* epistemic thesis, which picks out real ontological structure between detection-property-bearing entities. How can infinite-duration concrete structure (harmonic structure) be construed as real? Does a big-bang universe render infinite-duration optical structure impossible?[37] Rather than answer these questions, theorists might relegate harmonic structure to an idealization "which no one takes to have counterparts in reality" (Chakravartty 2017, 6), regardless of how useful that idealization is to optical theorizing, explanation, representation, and measurement. I argue in this section that idealizing the harmonics indispensable to the definition of intrinsic dispositional refractivity is unavailable to the semirealist, according to the semirealist account of idealization, and a similar account by Brian Ellis (1992).[38]

Firstly, I do not deny that Fresnel and his contemporaries probably employed harmonics only as explanatory devices, and not as denoting real existents.[39] As someone today might argue, Fresnel's (1900) appeal to infinite distance L (see section 3.3.2 above) explains diffraction without sanctioning actually infinite distances, because appeals to infinite L afford the kind of counterfactual comparison widely thought to facilitate

[37] Tentative support for a negative answer to this question appears, in my opinion, in Morganti (2013, 179) and Barrow (1998, Chapter 6). The controversiality of harmonic realism is otherwise evidenced by Weatherall (2014), Liston (1994), and Sheldon (1985).

[38] Many accounts of idealization exist, but to analyze their compatibility with semirealism falls outside the scope of this paper. I highlight Ellis's account for its electromagnetic dispositional realist subject matter.

[39] Kipnis (1984) explains that the principle of interference for light "was first accepted by a group of influential scientists simply as a means to give a mathematical treatment of diffraction," and that even for Fresnel, such "acceptance did not necessarily imply the support of the wave theory . . ." (223). The "wave theory" referenced here is that which appeals indispensably to harmonics.

epistemic explanation (Potochnik 2017; Saatsi and Pexton 2013; Woodward 2003). If L were infinite or sufficiently large, the interventionist argues, then the brightness differential between slit diffraction and single-edge diffraction would reverse, rendering variable L explanatory.[40] Theorists from Fresnel's day to the present also undoubtedly appreciate the simplicity of characterizing signals of *any* duration with a universal equation like Equation 1 or 2. Granting that Fresnel's laws literally apply only to harmonics may amount to nothing more controversial than granting that Fresnel's laws *approximate well* the refraction of all finite-duration signals.

For the semirealist's metaphysically weightier task of ascribing *intrinsic* dispositions, however, explanatory and representative considerations (of the previous paragraph) appear not to suffice. If my arguments of section 3.1 are tenable, then approximation and explanation do not buy us intrinsicality, only real harmonics buy us intrinsicality; and "intrinsicality" plays no prominent role, from what I have seen, in Fresnel's descriptive work. Hence what is the role of idealizations, if any, in the semirealist project of intrinsic property ascription?

3.4.1 Chakravartty on Idealization and Intrinsicality

The first point to note is that Chakravartty says very little about *infinite* idealizations.[41] He mentions the "infinite populations" posited in genetics research "to eliminate the effect of random drift . . . on gene frequency calculations" (Chakravartty 2017, 181), but this use of infinitudes appears explanatory like Fresnel's light source at L, rather than property-realistic or metaphysical, since calling gene frequencies "intrinsic"

[40] Subject to specific conditions outside my purposes to elaborate.
[41] For recent discussion on infinite idealizations in physics, see Shech (2018).

to populations (which Chakravartty does not do) would sound strange. On the other hand, Chakravartty (2013) explicitly unites the infinite with the intrinsic in is his thought experiment suggesting *how we could know* that a charged particle possesses its charge intrinsically: namely, if it retained its lawful structural relations with "a test charge at infinity, and then very close by . . ." (44).[42] The insight to glean from this thought experiment, I reckon, is that if both infinitely distant and highly proximate influences are equally lawful, then those external influences make no salient conceptual or metaphysical difference to the charged particle's possession of charge. The particle appears charged irrespectively of its lawful relations with other entities, or intrinsically charged (Chakravartty 2013, 43).

Chakravartty (2013) quickly adds, however, that this insight about intrinsicality gained *via* an infinite idealization is only epistemic; he calls it question-begging to infer that the "relations manifested in [thought] experiments somehow 'make' charge the property that it is . . ." (44). In a similar vein, the semirealist might reject my positing real harmonic structure to make refractivity (intensity) the property that it is. What this objection overlooks, however, is that positing intrinsic charge need *not* reify the infinite structure of Chakravartty's thought experiment, because *eliminating* that infinite structure does not render the charge concept viciously regressive, meaningless, or incoherent, as contrarily happens to the "per wavelength" optical property of refractivity (intensity) when its harmonic structure is eliminated, approximated, or nominalized (section 3.1). I avoid begging ontological questions for infinitely-sized structure, because I possess an

[42] Chakravartty (2013) is responding to a thought experiment put to him by French (2013, 7).

argument about the conceptual coherence of ascribing real intrinsic dispositions (section 3.1), not merely an epistemic argument about how we might come to know about them.

The deeper peculiarity about semirealism is why, in the process of ascertaining what ontology a concrete structure "minimally demands" (section 2), we are supposed to exercise an *elevated* caution when ascertaining from thought experiments what the *intrinsicality* of a detection property minimally demands.[43] Is speculation about which detection properties—and their interactions with scientific instruments—constitute the relata of mathematical laws, *different in principle* from speculation about how those detection properties could be "intrinsic"? A cursory glance suggests not, since one detects the charge from infinity with the same instrument as for the proximate charge (Chakravartty 2013, 44). Whether we are "detecting" properties and entities in an epistemic versus ontological sense[44] appears sometimes ambiguous within semirealism, but this question is too large to answer in this paper.

The main point is that Chakravartty (2007) clearly takes idealizations to be "descriptions of properties and relations that do not and cannot exist as described in any circumstances" (223), and so I cannot chalk up harmonics to a semirealist idealization in any obvious sense. He does offer an analogy whereby idealizations, although false descriptions, can refer to existents, as unrealistic artistic representations sometimes denote real objects or events in history (226). But I fail to see how the artwork analogy

[43] Perhaps this apparently elevated caution for ascertaining the ontological ramifications of intrinsicality is justified because intrinsicality is a metaphysical notion not itself expected to port between scientific theories. I have not seen Chakravartty suggest this claim, but it bears considering.

[44] By "ontological sense," I mean the sense in which a given property would be "intrinsic" to an entity.

applies to "per-wavelength" dispositional refractivity (intensity). By my arguments of section 3.1, the idealized harmonic "artwork" in question would refer to something which could not exist at all, pulse-refractivity, which is like saying that a given painting denotes a non-cubical cube. I am hesitant to analogize a property so fundamental to science (refractivity) as an artwork so nihilistic and abstract.

3.4.2 Brian Ellis (1992) on Idealization and Intrinsicality

Chakravartty's (2013, 44) example of the charge coming in from infinity strongly resembles Brian Ellis's (1992) account of idealizations in science, and because Ellis and I come to *exactly opposite* conclusions about the ontology of the Fourier harmonic in electromagnetic dispositional property ascription, Ellis's account merits examining for the present discussion. In short, Ellis encourages philosophers to utilize idealizations in science, because their falsity or inaccuracy has no bearing against the aims of science. In Ellis's words, "[t]he aim of science is not to describe what actually happens in nature . . . . it is to *explain* what happens" by appeal to intrinsic properties (266). While I do not attribute these claims to any semirealist, Ellis's approach to ascertaining intrinsic properties is likely to capture the attention of semirealists.

For just as in Chakravartty's (2013, 44) charge-from-infinity thought experiment, Ellis (1992) advocates discovering intrinsic properties by abstracting from external influences (272). To explain X-ray "diffraction patterns,"[45] for example, and to understand "the nature of X-ray diffraction" (275), a nature constituted by the intrinsic properties (272) of a metallic or crystal lattice, Ellis surmises that we need to idealize

[45] X-ray diffraction is analyzable by the same mathematical and physical principles as those used to study Fresnel's laws.

away from certain realities which are “enormously complex,” including the reality that propagating X-ray waves “will not be monochromatic . . .” (276). That is, Ellis *assumes* that we must treat monochromatic waves as an idealization in order to ascertain the intrinsic properties of media that drive X-ray diffraction. In criticizing this assumption, I am not implying that Ellis has overlooked any recommendations from *science* about the existence of monochromatic waves; quite the opposite (see my footnote 15). I criticize instead his implicit assumption that the metaphysically serious enterprise of intrinsicality can be achieved or ascertained through the metaphysically (ontologically) unserious enterprise of idealization. However well that assumption works for properties besides per-wavelength electromagnetic properties, my arguments of sections 3.1-3.2 suggest that *immediately* idealizing what science would idealize can cost the metaphysician her intrinsic property ascription altogether.

Here someone might object (again) that I commit the flagrant fallacy of reifying, without argument, that which turns out to be conceptually necessary. Even if it turned out that the cube was the only shape by which to model the color solid for human vision (*viz*. to model the dimensions of hue, saturation, and lightness that we see), that fact would not by itself prove that any cubes exist.[46] Likewise, even if harmonics were indispensable to intrinsic refractivity ascription, that fact would not prove that harmonic structure exists. I can easily reply, however, that in a world with no cubes, no one would walk around saying, “*There* is an instance of the color solid,” or “*There* is a cube!” People would instead say, “There is a drawing of the color solid [or of a cube].” The semirealist, on the other hand, *is* pointing out “intrinsic” properties like intensity

[46] I thank Matteo Plebani for this color-solid objection.

(refractivity), and if my arguments of section 3.1 hold, then this practice appears no less innocuous than reifying entities out of conceptual necessity, since pulse-refractivity is not a thing at which anyone could point.

Said another way, I am not reifying harmonics out of conceptual necessity, because I am instead respecting the antecedent intrinsicality constraint of semirealism. The burden is on the semirealist to accept or reject the infinitely-sized structure entailed by her intrinsicality constraint about the detection properties that she calls real, and this burden is her semirealist dilemma. Ellis's (1992) account of electromagnetic properties[47] founders on the intrinsicality horn of that dilemma, because he demands intrinsicality for those properties, but idealizes the monochrome harmonics that could give him intrinsicality. Semirealists should (and must) avoid this mistake.

4. Conclusion

In this paper I proposed a dilemma for semirealism, namely that the structural implications of calling dispositional intensity "intrinsic" to light, or refractivity "intrinsic" to media (as semirealists do in the Fresnel-Maxwell case study), undermine the very dispositionality of those properties. Their dispositionality is undermined because their intrinsicality entails that they manifest structure of infinite temporal extent (section 3.1), and an always-manifesting disposition is not the characterization that semirealists (and other philosophers of science) typically give for per-wavelength dispositional properties like reflectance, refractivity, diffraction, etc. I argued that semirealism can retain the intrinsicality and dispositionality of optical properties by

[47] Namely the reflectance and refractivity responsible for X-ray diffraction (Ellis 1992, 276).

reconstruing what is meant by a “dispositional” detection property (section 3.3.1), but I denied that calling the infinite-duration structure an “idealization” will work in the present semirealist account (section 3.4.1), or within the similar account of Ellis (1992) (section 3.4.2).

Due to semirealism’s leeway about what counts as a “detection property,” and due to the limited scope (so far) of the semirealist dilemma, I doubt that it seriously threatens semirealism, or that my arguments will change many realist or antirealist convictions. My arguments instead reveal that realist approaches to ascribing intrinsic properties, and especially to idealizing as part of that process, have sometimes been too quick, and can turn out to be self-defeating. For the infinitely-sized structure that I identify for the semirealist account of Fresnel-Maxwell refraction is nothing new to the work of *Fresnel* and his contemporaries (section 3.3.2), but the intrinsicality requirement of semirealism renders this structure more real than either Fresnel or many contemporary researchers would countenance; and that new ontology upsets the dispositionality of refractivity altogether.

CHAPTER 4

A COUNTEREXAMPLE TO DEFLATIONARY NOMINALISM*

1. Introduction: Azzouni and Colyvan on mathematical realism

In the debate between Platonists and nominalists about mathematical ontology, Jody Azzouni (2012b; 2010; 2009; 2004a; 2004b) defends a "deflationary nominalism"; deflationary in that mathematical sentences are true in a non-correspondence sense,[1] and nominalist because mathematical terms—appearing in sentences of scientific theory[2] or otherwise—refer to nothing at all. In this paper, I focus on Azzouni's positive account of what *should* be said to exist. The quaternary "sufficient condition" (Azzouni 2004b, 384) for posit[3] existence, Azzouni (2012b, 956) calls "*thick epistemic access*" (hereafter TEA), and in this paper I argue that TEA surreptitiously reifies some mathematical entities. The mathematical entity that I argue TEA reifies is the Fourier harmonic, an infinite-duration sinusoid applied throughout contemporary engineering and physics. The Fourier harmonic exists for the deflationary nominalist, I claim, because the harmonic plays what

*Submitted to *Erkenntnis* on December 26, 2020.

[1] More precisely, Azzouni's deflationism interprets truth as nothing above and beyond the "generalization" expressed by the Tarski biconditional (e.g.): "Snow is white" is true iff snow is white (Azzouni 2010, 19). Hence what redeems that biconditional, in Azzouni's account, is neither strictly correspondence, nor coherence, nor indispensability of the truth idiom to language. On the other hand, Azzouni rejects truth pluralism (see Azzouni 2010, §§4.7-4.8). The best articulation of Azzouni's deflationary account of truth in science, mathematics, and applied mathematics may be Azzouni (2009), but see also Azzouni (2010, Chap. 4). The details will not concern me in this paper.

[2] Azzouni (2014) understands scientific theories to be "linguistic entities" (2995) "written in natural languages supplemented with additional technical vocabulary" (2994).

[3] Posits are the alleged referents of singular language terms.

Azzouni calls an “epistemic role” (see section 2) in the commonplace observation of macroscopic entities, for example in viewing a vase with the human eye. Thus I present a counterexample to deflationary nominalism, from assumptions that the deflationary nominalist holds or should accept. I support this counterexample by a positive argument, what I call a “second way” (the first way being Azzouni’s) to ascribing a posit an epistemic role.

Mark Colyvan (2010) has already criticized TEA for admitting existent mathematical entities,[4] but he argues from an assumption that the deflationary nominalist denies on independent grounds (as Azzouni 2012b, 962-963 rightly objects). I avoid Colyvan’s objectionable assumption, but before explaining how, it helps to understand a bit of Azzouni’s terminology, which has developed somewhat over the years (cf. Azzouni 1994). The posits referenced by singular language terms come in three varieties for the deflationary nominalist: “thick” (e.g., elephants or molecules that someone has detected with their senses or instruments), “thin” (e.g., elephants that no one has encountered but has reason to believe exist due to scientific theory), and “ultrathin” or referentially empty and not existing in any sense (Azzouni 2004a, 128-129). Azzouni treats mathematical entities as ultrathin.

Colyvan (2010) argues straightforwardly against this taxonomy. Specifically, he appeals to Azzouni’s (2004a, 138) criteria for thin-posithood. Those criteria were[5] that the posit exhibit the “Quinean virtues” of “simplicity, familiarity, scope, fecundity, and

[4] By an argument different from his influential contributions on explanatory indispensability (Colyvan 2001).

[5] Azzouni (2012b; 2004b) has developed his account of posit existence to focus on TEA conditions (see section 3 below), de-emphasizing the Quinean virtues mentioned in this sentence of the main text (and prevalent in Azzouni 2004a).

success under testing" (128) in scientific theory, and that there be a "defeasibility condition" or reason that the posit could not be thickly[6] detected. Colyvan (2010) argues that mathematical entities fulfill the Quinean virtues, and that their "'excuse clause'" (i.e. defeasibility condition) for eluding detection is their abstract nature (288).[7] In reply, Azzouni (2012b) rejects Colyvan's excuse clause as "philosophical" and not "scientific" (963), since Azzouni thinks that thin-posit discriminations should hail *from science* (962). He additionally cites an independent reason for doubting the existence of mathematical abstracta, an argument that he calls the "epistemic role puzzle" (hereafter ERP; 963, footnote omitted), which I discuss in section 2. Thus Azzouni (2012b) takes Colyvan's (2010) mathematical reification attempt to fail.

My argument for a limited mathematical realism differs from Colyvan's (2010), in that I attempt to meet the demands of the ERP as they have been codified into TEA criteria. Whereas Colyvan's "philosophical" excuse for thin-posithood appeals to the allegedly abstract nature of mathematical posits, I analyze the function of mathematical posits within TEA. I argue that some mathematical posits *qua mathematical*—*viz*., in a sense different from spatiotemporal abstractness[8]—prove indispensable to achieving TEA, or to forging[9] TEA to a "thick" posit like a vase by ordinary visual perception.

[6] The "thickness" of epistemic access tracks the "thickness" of the posit accessed, such that only "thin" access would be had to a thin posit (and no access to an ultrathin posit).

[7] Colyvan (2010) motivates this conclusion in a more nuanced and compelling way than I have summarized here, by appealing to "borderline" (290) cases of posit thinness that need not be elaborated for the present discussion.

[8] Mathematical characteristics differing from spatiotemporal abstractness include the "primeness" and "oddness" of 3. Azzouni (2009) agrees that "the symptoms of being mathematical" need not include "being 'outside of space and time' . . ." (165).

[9] Azzouni consistently uses this word to describe how an agent comes to stand in a "thick" epistemic relation to a posit (Azzouni 1997, 477, 480, 483; 2004a, 147, 150, 173;

More precisely, I argue that the infinite duration of the Fourier harmonic is a mathematical property (or dimension) indispensable to ascribing reflectance as a property of vases; reflectance being a property that Azzouni (2010, 30) alludes to obtain on the surfaces of vases, and to facilitate the forging of TEA to them.[10] Thus I attempt to commit the deflationary nominalist to the thin-posithood of a mathematical entity *before* the scientist has occasion to deny its existence.[11] I also provide an excuse clause different from Colyvan's (2010) for why we cannot or do not thickly detect the Fourier harmonic that possesses an epistemic role.

Spelling out my argument requires some groundwork. Section 2 reviews the ERP and its function as a premise alongside TEA in Azzouni's argument for deflationary nominalism. Section 3 then outlines the conditions for TEA, and section 4 presents my argument[12] for the indispensability of the Fourier harmonic to reflectance ascription. In section 5, I answer the ERP with respect to the Fourier harmonic, proposing a "second way" to an epistemic role (Azzouni's first way appearing in section 3). I also respond to the "coding" objection of Azzouni and Bueno (2016), which despite my "second way," would nominalize the harmonic to ultrathin status. Section 6 concludes.

2009, 149; Azzouni and Bueno 2016, 813). "Forge" lacks a technical definition, and just means "establish" or "achieve."

[10] In recent work, Azzouni (2017; 2012a) explicitly rejects property realism. My arguments of this paper remain relevant, however, because the recent Azzouni (2017, Chap. 8) endorses TEA without clearly extirpating property reference within TEA. As I explain in due course, removing property reference from TEA is no trivial matter.

[11] Some scientists like Michael Redhead (1988) already doubt the existence of Fourier harmonics, so my use of "before" in the current sentence of the main text refers to the theoretical procedure of deflationary nominalism, and not to the chronological history of our world.

[12] Elaborated in Danne (2020).

2. The Epistemic Role Puzzle and Thick Epistemic Access

Pivotal to Azzouni's deflationary nominalism about mathematical entities, and to its taxonomy of thick, thin, and ultrathin posits, is his "epistemic role puzzle" (ERP):[13] the ostension that numbers play no "epistemic role" in mathematical practice, in contradistinction to the entities of realist science that play an epistemic role in scientific practice. On what an epistemic role amounts to, Azzouni (2016) is the most explicit:

> the official concern of the puzzle is this: notice that our standard *epistemic practices* have certain accompaniments: methods of recognizing the *epistemic artifacts* that our means of access to the objects in question have *because of* those means of access. (Azzouni 2016, 12)

"Epistemic artifacts," in Azzouni's account, "are the ways that our means of access to objects distort our impressions of the properties of those objects" (5), for example the way that squinting one's eyes (Azzouni 2004b, 383) increases the optical resolution of an object's surface. Thus unlike Benacerraf's (1973) Dilemma against Platonism, which asks how we can possess mathematical knowledge despite the acausal nature of mathematical abstracta, the ERP asks why mathematics lacks "an ancillary science" that investigates mathematics' own epistemic artifacts (Azzouni 2016, 12), a question that pertains even if mathematical objects are *not* abstracta.[14]

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[13] Discussed in several works, including Azzouni (1994, I, §7; 2000; 2010, §1.3; 2015; 2016).

[14] McEvoy (2012) contends that the ERP reduces to Benacerraf's Dilemma unless the ERP is conjoined with premises that render the ERP redundant. Azzouni (2016) convincingly counterargues that McEvoy overstates his case. The debate does not affect my paper, which focuses on the workings and conditions of TEA rather than those of ERP, although my focus on TEA provides a "second way" of answering the ERP.

Azzouni employs the ERP and TEA as premises for deflationary nominalism. The argument[15] (what I call the "Ultrathin Mathematics Argument") can be paraphrased as follows:

Criterion: "anything that exists is mind- and language-independent"[16]

TEA: "*we* recognize that an object is mind- and language-independent" when "it has an *epistemic role*"[17]

ERP: mathematical entities lack an epistemic role

Conclusion: mathematical entities do not exist

Azzouni (2016) acknowledges that the Ultrathin Mathematics Argument may appear to prove only that theorists lack "'*reason to believe*'" in mathematical entities (10), and not that they do not exist. He urges the stronger **Conclusion**, however, for the same reason that we do not say (without strain) that we lack reason to believe in "hobbits" or in "Santa Claus"; we instead "say" with aplomb that hobbits and Santa do not exist (10).[18]

This appeal to language use, and to what we *say*, to derive the defeasible[19] ontological **Conclusion** from the epistemic premise **TEA** follows, in my view, from Azzouni's "linguistic" arguments for mathematical nominalism more generally (Azzouni 2015, 1149). That is, while I must pass over them in this space, I accept for the sake of

[15] Summarized in Azzouni (2016, 9-10).
[16] Text quoted from Azzouni (2016, 9).
[17] Text quoted from Azzouni (2016, 9-10).
[18] See Azzouni (2015) for arguments that we should not be "agnostic" about mathematical ontology, for a somewhat different reason.
[19] Azzouni (2016, 10): "surely the fact that I've no reason to believe in [hobbits] is compatible both with my being able to draw the conclusion: there are none of these things *and* I might be wrong about this."

discussion Azzouni's extensive efforts to show that fictional characters like hobbits and Santa Claus exist in no sense at all,[20] and that "there is" in the vernacular fails to pick out hobbits or numbers in first-order regimented theories (Azzouni 2004a, Chapter 3). To reiterate my disclaimer from section 1, I accept Azzouni's independently argued accounts of deflationary truth and natural-language science that render the Ultrathin Mathematics Argument more cogent than I have outlined it. I argue instead that the constitutive principles of the **TEA** premise, which I list in the next section, falsify the **ERP** premise with respect to the Fourier harmonic, and thus falsify the **Conclusion** of the Ultrathin Mathematics Argument with respect to the Fourier harmonic. The applicability of my argument to mathematical entities besides the Fourier harmonic is a topic for another occasion.

3. Thick Epistemic Access and Ordinary Visual Perception

In explaining the epistemic role of posits generally construed, Azzouni contrasts two examples in a passage worth quoting at length:

> Should *S* see an urn, and think, "that's an urn," crucial to his thought being about *that urn* are (nonconceptualized and nonrepresentational) facts about perception that are (at least partly) involved in the relationship between *S* and the urn. One therefore cannot simply replace the urn with a vase in a thought experiment (corresponding to the referential-order thought experiment above about $\underline{1}$, $\underline{2}$, $\underline{3}$ . . . and 1, 2, 3 . . .), and have everything go swimmingly. The relationship between *S* and that urn is based partly on the perceptual interactions between *S* and that urn. It's those perceptual interactions that indicate (in part) "the epistemic role" of the urn itself [. . .]. For when we engage in a detailed study of the perceptual abilities of *S*, what emerges is a description of—to put it roughly—the sorts of things *S* is capable of distinguishing by perception (and why). At this point, the actual (and perhaps dispositional) properties possessed by the *urn* become relevant [. . .]. (Azzouni 2010, 30)

[20] Azzouni (2010, Chapters 1 and 3; 2004a, Chapter 3).

The number puzzle (1, 2, 3 . . . and 1, 2, 3 . . .) referenced in this passage illustrates the ERP. If the alleged referents of numerical terms were "swapped" clandestinely, then mathematical practice would allegedly proceed unabated, in a way that the study of urns could not proceed if they were swapped with vases. I have already directed readers to auxiliary debate about the ERP in footnote 14. Here I focus on Azzouni's passing but implicatory remark about property ascription, namely that some properties of the vase[21] facilitate perception of it, and thus partially constitute—as I will explain in this section—the TEA forged between the human perceiver and the vase. I will eventually argue that the Fourier harmonic plays an epistemic role *via* the ascription of dispositional reflectance[22] to the vase, a property that renders the vase perceptible.

Reflectance is a good candidate property responsible for the vase's perceptibility,[23] for two reasons. The first is Azzouni's (2005) implicit concession that reflectance could play a role in color perception (101-102, 105), although he doubts that color *reduces* to reflectance as a natural kind (105). The second reason follows from the first, namely the established philosophical pedigree of reflectance in perceptual theory (Byrne and Hilbert 2003; Jackson 1998; Hilbert 1987), despite ongoing controversies about whether color reduces ontologically to reflectance.[24] Thus to be clear, my thesis

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[21] Azzouni (2010, 30) discusses both urns and vases, but I focus on vases for their familiar role in philosophical discussions of another "dispositional" property: fragility (Schrenk 2017, §3.1).

[22] I say "dispositional" because Azzouni (2010, 30) does. My argument applies equally to categorical renderings of reflectance, like Frank Jackson's (1998, Chap. 4; 1996). On the difference between dispositional and categorical properties, see Schrenk (2017, Chapter 2).

[23] And not exclusively so; a perceptible vase must also possess a "shape," "mass," or "surface," perhaps.

[24] A recent critic of this reduction is Gert (2017, Chapters 1 and 3).

has nothing to do with whether human-visible colors plausibly reduce to sets of reflectances.[25] I claim only that reflectance is ostensibly a surface property to which radar systems respond, and a property that conditions much of the ambient light striking the human retina; thus reflectance likely occupies a role within TEA, whether TEA be forged by the human visual system, or by a radar system, etc.

Before listing the conditions of TEA, to show how reflectance fits among them, it pays to recall the deflationary nominalist's **Criterion** for posit existence (section 2). While TEA is a sufficient condition for posit existence (Azzouni 2004b, 384), **Criterion** is a necessary condition: for a posit to exist, it must be "mind- and language-independent" (Azzouni 2016, 9). A clear antonym of mind- and language-independence, in Azzouni's (2012b) account, is the quality of being *stipulated* (955).[26] Thus any posit's properties that facilitate TEA should be non-stipulated to obtain at or on the posit, and the empirically defeasible[27] necessary conditions of TEA, which I sometimes call "ingredients," go some way toward precluding such stipulation:[28]

1) **Robustness:** Properties or entities observed can diverge from what or how a theory predicts them to be, or from what observers "believe about what they'll observe." Alternatively: "what instruments detect greatly outstrips what theories predict" instruments to detect (383).

[25] This reduction is defended by Byrne and Hilbert (2003), and Hilbert (1987).
[26] For example, Azzouni (2004a, 56-57) holds that the properties of Mickey Mouse are stipulated, not discovered. Hence the fictional character Mickey exists "in *no sense at all*" (57).
[27] Azzouni (2012b, 956-957): "It's an *empirical claim* that the only way we have to discover anything about ontologically-independent objects involves epistemic processes that must include appropriate sensory or instrumental interaction either with those objects, with objects they have affected, or with other suitably theoretically-related objects."
[28] I paraphrase the following list from Azzouni (2004b, 383-384), quoting where appropriate. The boldface titles of the four conditions I take from Azzouni (2004a, 129).

2) **Refinement:** "[T]heory-free" means exist for "adjusting and refining observations" (383), or for "adjusting and refining instruments and what they reveal . . ." (384). Such theory-free methods just are those pre-scientific methods by which we discern various regularities in the world, such as by "squint[ing]" our eyes (383).

3) **Monitoring:** "What's observed can be monitored . . . over time . . ." (383).

4) **Grounding:** "Certain properties of the object observed can be used to explain why, and in what respects, observed things can be observed" (383). That is, we can "study . . . how the instrumental access to items reveals properties of what's being studied" (384, footnote removed).

One may notice that the term "properties" appears in the first and fourth TEA ingredients. Posits are **robust** if they or their properties exhibit characteristics that surprise theorists, or if such surprises are possible in principle. Properties also **ground** the existence of a real posit, by providing a reflexive mechanism by which an observer can discern that properties of the posit render the posit observable.

Colyvan explains **Grounding** in two helpful passages. In the first, he says, "I can tell that a jet is moving across the sky by observing its vapour trail and seeing that the leading edge of the trail is advancing across the sky" (Colyvan 2005, 221). The non-stipulated reflexivity here is between the object that produces vapor trails (perhaps for theoretical reasons believed on independent grounds), and the dynamicity of the vapor trail that indicates objectual movement. Another example is Colyvan (2010, 288): "we can identify the heart in a chest x-ray because its relative density means that it appears as a region of greater x-ray absorption and this, in turn, enables us to determine other properties of the heart, such as its size." Here the reflexivity is between the heart's density and the kind of instrumental procedure (x-rays) that distinguishes the heart from other, non-heart objects.

My argument to bestow upon Fourier harmonics an epistemic role by ascribing reflectance to a vase, an ascription which reifies Fourier harmonics as thin posits, exploits both the **Robustness** and **Grounding** ingredients of TEA. Before laying out that argument, however, I must first motivate the deflationary nominalist *to apply* the Fourier harmonic in reflectance ascription at all. I attempt that motivation next.

4. The Mathematics of Ascribing Dispositional Reflectance

4.1 A metaphysical problem with ascribing reflectance

Harmonic realism follows, in deflationary nominalism, from a particular metaphysical problem[29] with ascribing reflectance as a real property of surfaces. The problem is that the most philosophically accessible definition of reflectance (Hilbert 1987; Byrne and Hilbert 2003) happens to be an operational definition:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance [SSR] of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987, 1037-1041)[30]

This definition might not appear operational at first glance (quite the opposite, considering its language about "stable properties of objects"), but in this section I shall argue that Hilbert's definition (hereafter "pulse-SSR" or "Hilbertian SSR") functions only as an operational definition of reflectance, and that the attempt to ascribe Hilbertian

[29] Detailed in Danne (2020).

[30] Hilbert describes the reflectance ratio somewhat infelicitously in this passage, suggesting that incident flux comprises the numerator of the ratio; that insinuation is wrong, and inconsistent with the rest of Hilbert (1987); the SSR ratio is reflected/incident flux.

SSR as a property *of vases* fails. Only a reflectance defined in terms of Fourier harmonics (which Hilbert's definition lacks) can be ascribed to vases and other surfaces.

Here's why. Firstly, "flux" in Hilbert's definition means average power in watts (Hilbert 1987, 1033-1042; cf. Germer et al. 2014), but only in the colloquial sense that "average power" could *be* nonzero for finite-duration pulses of light. Signal theory, which omits the colloquialism employed by Hilbert and some spectrophotometrists,[31] permits only *infinite-duration* signals to possess nonzero average power, since the energy of the signal for which average power is computed is itself an integration over infinite time (Haykin and Van Veen 1999, 20-21).[32] By this crucial difference, signal theory accounts for a classical (non-quantum) behavior of light (Hirlimann 2005, 31) that undermines the attempt to ascribe pulse-SSR to surfaces. That behavior of light I call "harmonic dispersion," the inverse relationship of a pulse's bandwidth to its duration, or the ubiquitous, empirical, and well-documented tendency of monochromatic light to become heterochromatic as its pulse duration decreases (Stingl et al. 1995; Deng et al. 2005).

[31] To be clear: calculating the "average power" of finite-duration signals is a very common and useful practice in science and engineering, but a woefully bad practice for metaphysicians to adopt, as I argue in this section.

[32] That is, signal theorists classify any signal possessing an "average power" to also have infinite energy (Haykin and Van Veen 1999, 21).

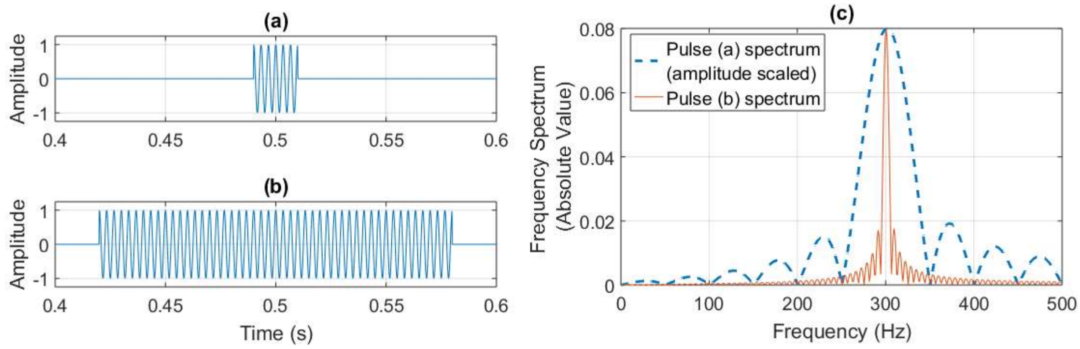

Figure 4.1: Harmonic Dispersion, 300 Hz

Figure 4.1 illustrates harmonic dispersion at a carrier frequency (300 Hz) far below that of light (for ease of modeling), nevertheless indicating the dramatic dispersion (c) that occurs at all carrier frequencies. The point to notice is that pulses (a) and (b) differ only in their durations, but exhibit a considerable difference in their bandwidths (c). Hilbert's definition rightly allows a surface's reflectance profile[33] to possesses different values at different wavelengths, but only pulse (b) propagates at anything close to one wavelength (i.e. 300 Hz);[34] pulse (a) propagates as a wide "envelope" of wavelengths, per the dotted line in (c).

Thus harmonic dispersion generates what I call the "Vicious Reflectance Regress" (VRR) against Hilbertian SSR. This regress refers to the precipitous collapse of the pulse-SSR value alleged to obtain at a surface, for a given wavelength. Assume, for example, an optical 5 W pulse centered[35] at 800 nm. Empirical data confirms that when the duration of such a pulse falls below 1 picosecond (ps; 10^{-12} seconds), the pulse's

[33] A "reflectance profile" is the set of (or a plot of the set of) a surface's reflectance values (between 0 and 1) across the human visible range of wavelengths; for an example, see Byrne and Hilbert (2003, 9, Fig. 1).
[34] "Wavelength" and "frequency" are interchangeable terms in applied optics. Frequency is speed of light divided by wavelength.
[35] As the plots of Figure 4.1 (c) are "centered" at 300 Hz.

bandwidth grows as wide as 100 nm (Deng et al. 2005; Stingl et al. 1995). Thus, if this pulse propagates into a perfectly reflecting mirror (SSR = 1 at all wavelengths), what is the "average power" of light expected to reflect at 800 nm? The pulse-SSR theorist ostensibly needs to answer: 5 W. But I argue that the answer cannot be 5 W, since the original 5 W pulse is spectrally redistributed by harmonic dispersion (Figure 4.1).

Assume very roughly, then, that only 80% of the 5 W pulse actually reflects at 800 nm (the remaining 1 W dispersing to neighboring frequencies). Can we say that the average power of this reflected, 800 nm *component* of the original pulse is 4 W (80% of 5 W)? Again, I say no, because that 4 W "component" is itself a finite-duration pulse (what else could it be?), and *ex hypothesi*, pulses disperse their frequency content when they are short-duration. Thus the 4 W pulse really only propagates with 3.2 W (80% of 4 W) at 800 nm, *ad infinitum*. By inspection, this regress obtains no matter what dispersion percentage is originally picked,[36] and this regress is vicious, destroying the pulse-SSR property by rendering it conceptually incoherent, and by asymptotically driving any given pulse's per-wavelength "average power" to zero.

The VRR renders pulse-SSR conceptually incoherent because there is no principled stopping place within the infinite iterations (5 W, 4 W, 3.2 W, etc.) to construct the SSR ratio (of reflected and incident average powers—see Hilbert's definition opening this section). Nor does stopping the regresses for the numerator and denominator of the SSR ratio at the *same* iteration solve anything, since one can still ask

[36] This point matters, because all finite-duration pulses are dispersive, even the hyper-picosecond pulses. Their harmonic dispersion might not be as radical as Figure 4.1 (c), but will still obtain, and so perhaps a 99.999% regress ensues, which is nevertheless just as vicious as the 80% regress discussed in this section.

why the arbitrary stopping point was picked, and why every given pulse's per-wavelength average power plunges to zero. Nor can one appeal to the "average power" that was "actually measured" in a laboratory to stop the regress; that proposal begs the question against the VRR, about which "pulse" in the regressive iteration was measured. No one knows. If the measurement device reads 4 W at 800 nm, one can justifiably *ask* why the value is not 3.2 W, and conversely; whatever is being measured by the device is not a *pulse*, by the argument just given. "Per-wavelength pulse-reflectance" is a contradiction in terms, like "non-cubical cube."

Philosophically, then, I conclude that pulse-SSR cannot be a property of vases, because ascribing to vases a property that is viciously regressive and conceptually incoherent is disingenuous at best, and vacuous at worst. Pulse-SSR can only be an operational property, a set of instructions about how to measure the colloquial "average power" ratios of long-duration, (relatively) non-dispersive reflecting signals; pulse-SSR is not sufficiently well-defined to be a property of the vase.[37,38]

4.2 A solution to the Vicious Reflectance Regress

Blocking the VRR is where the Fourier harmonic comes into play. Fourier harmonics just are the infinite-duration signals that signal theorists use to compute average power, and which I have already represented in Figure 4.1 (c). Every point in either trace of Figure 4.1 (c) represents a harmonic possessing that trace point's plotted

[37] Nor is pulse-SSR sufficiently well-defined to be a property of the vase-and-impinging-light, an "extrinsic disposition" (Hoffmann-Kolss 2010; McKitrick 2003). The VRR militates against any definition of reflectance employing per-wavelength "pulses."

[38] See Danne (2020) for additional objections, with replies. Note that the wave-particle duality of light does not affect my argument, because photonic emission and absorption also occur "per wavelength" in finite time, launching the VRR.

amplitude and frequency. All of the harmonics represented in a Figure 4.1 (c) trace superimpose without remainder, moreover, into that trace's corresponding pulse (a) or (b).[39]

The philosophical point to grasp is that due to their infinite duration, harmonics never disperse their frequency content; they are *immune* to harmonic dispersion, and they possess unity bandwidth, by definition. Thus reflectance redefined as the per-wavelength efficiency of a surface to reflect harmonics is conceptually coherent and ascribable, since it never suffers the Vicious Reflectance Regress. No matter the duration of the "pulse" propagating into a mirror, the per-wavelength reflective efficiency of that mirror remains a stable, constant, and well-defined property if that property is harmonic-SSR. When the laboratory measures 4 W at 800 nm, it measures the average power of a finite-duration portion of a *harmonic* that is not cancelled-out in superposition by other harmonics (Figure 4.1). Thus the regressive chase to measure the 800 nm power of a "pulse" never begins. While mathematical representations besides Fourier analysis can model electromagnetic pulses (e.g., wavelets, Bessel functions), the philosopher ascribing "per-wavelength" reflectance to surfaces as a real property appears to have a particular need (due to the VRR) of signal components that are "per wavelength" in the most literal sense. The harmonic, a monochrome, is that very signal.[40]

In the next section, I argue that the Fourier harmonic possesses an "epistemic role" in the deflationary nominalist sense; not for the harmonic's utility in predicting or

[39] The Fourier composition includes harmonics of negative frequencies, and requires a wavelength-dependent phase shift, neither of which are shown; this detail does not affect my argument.

[40] Many if not all wavelet bases, on the contrary, are heterochromatic, and will introduce dispersion effects (Deng et al. 2005; Mallat 1999, 546-547).

representing harmonic dispersion (as in Figure 4.1), but for its indispensability to ascribing reflectance to vases, and so of forging TEA to vases. Nominalize the harmonic by "approximating" it away from its infinite duration, my argument goes, and the Vicious Reflectance Regress ensues, destroying reflectance ascription to the vase, and disrupting the TEA alleged to obtain between the observer and the vase.

5. My Philosophical Excuse for Mathematical Realism

5.1 A second way to acquire an epistemic role

I am now in the position to argue that the Fourier harmonic possesses an "epistemic role" in the deflationary nominalist sense (section 2), because the harmonic proves indispensable to rendering dispositional reflectance ascribable to vases. That is, the harmonic proves indispensable for ascribing the reflectance property (section 4) that partially constitutes TEA to vases. Granted, a harmonic does not need to *exist* as the form of propagating light for harmonic-SSR to be ascribed; the harmonic needs to exist if harmonic-SSR ever incurs stimulation or manifests.[41] But some deflationary nominalists allude that vases do manifest reflectance (see Azzouni 2010, 30), and so the epistemic role—if any—of the harmonic should be scrutinized.

In effect, I am introducing a second way that a posit (the harmonic) could acquire an epistemic role. The first way was explained in section 2: a posit possesses an epistemic role if it produces "epistemic artifacts," if we can *tailor* our observational

[41] I hereafter imply rather than repeat this important point. Any reference that I make to an "ascribed" property is always a locution for "an ascribed property whose stimulus or manifestation (in a dispositionalist or non-dispositionalist sense) has occurred, is occurring, or is expected to occur." For my purposes, the stimulus of reflectance is light impinging on a surface, and the manifestation is light propagating away from the surface. Hilbert (1987) says relatively little about the stimulation and manifestation of SSR; cf. Boghossian and Velleman (1989); Jackson (1996); Byrne (2001); Pasnau (2009).

interaction with the posit. This tailoring receives explicit codification as the **Refinement** ingredient of TEA, but it seems fair to say that epistemic artifactuality includes the reflexivity, temporality, and surprise of the other TEA ingredients (section 3). Thus the traditional way to argue that the Fourier harmonic possesses an epistemic role is to identify its epistemic artifacts, and I will venture the traditional way in section 5.2; here I propose that a given *posit* can possess an epistemic role for an observer's knowledge of *another posit*.

Consider, for example, Colyvan's jet-vapor example of **Grounding** (section 3). One **grounding** property in the jet example is the vapor trail, since its spatial elongation is our defeasible indicator that a metal airplane exists as a thick posit. That the vapor trail exists can be inferred by its own epistemic artifacts: it looks a little different if we squint or hold up binoculars. But what property **grounds** the thick posit vapor trail? The answer is plausibly reflectance! By understanding reflectance, we can understand why different observational methods confirm the presence of water vapor, despite its possibly different appearances under those methods. Granted, Azzouni's (2010, 30) vase anecdote is a rare example of dispositional property ascription within the many of his references that I cite in this paper, and so I shall avoid suggesting that TEA can be forged to self-standing *properties* like reflectance, or that "properties" can be thick or thin posits.[42] That move would be metaphysical overkill for the deflationary nominalist. Instead, I point out that the jet plane kicks off epistemic artifacts *because* it generates vapor trails; a posit may generate epistemic artifacts through its **grounding** property. Hence my

[42] Especially considering Azzouni's (2017) antirealism about properties (despite his retention of unchanged TEA criteria); see section 5.4 of this paper for discussion.

question is: does not the property that *bestows* an epistemic role on a posit (as the vapor trail bestows that role on the jet plane) *thereby* possess an epistemic role of its own? How can one bestow what one lacks? My "second way" of ascribing an epistemic role to a posit, then, is by showing that posit to be the **ground** of TEA to another posit. The Fourier harmonic **grounds** the vase that we see, because the harmonic's infinite duration—a *mathematical* property in Azzouni's account[43]—(a) "explain[s] why, and in what respects" the vase "can be observed" (Azzouni 2004b, 383), as well as (b) how "instrumental access" to the vase "reveals" its properties (Azzouni 2004b, 384).

Clauses (a) and (b) in the previous sentence re-quote the **Grounding** ingredient of TEA (section 3). The Fourier harmonic satisfies (a) because it explains why the vase can be observed: because harmonic-SSR is what makes *the vase* reflective (section 4). The Fourier harmonic satisfies (b) because it explains how human vision or radar systems "reveal" the vase to be reflective: vision and radar work, ostensibly, because the vase possesses harmonic-SSR. Thus the Fourier harmonic possesses a **grounding** role, and so an epistemic role in perceiving vases. Such is my "second way" to ascribing an epistemic role.

Granted, the Fourier harmonic remains a "thin" posit because I did not claim to forge TEA to *it*, and so I need an excuse clause (section 1) regarding its undetectability. Unlike Colyvan's (2010, 288) appeal to spatiotemporal abstractness, I submit that the Fourier harmonic goes undetected because it propagates in zero-sum superposition outside the duration of the "pulses" that we take ourselves to manipulate (see Figure 4.1).

[43] Azzouni (1994) calls the "'infinite'" a "mathematical notion[] . . . [that is] not first-order definable . . ." (3).

An elaboration of the same insight is that even if we filter one frequency from a pulse with high precision, we finite beings cannot have "thick" access to its infinite duration *qua* infinite. Yes, Azzouni prefers that thin-posit discriminations hail from science (section 1), but I reply that according to my "second way" argument, Azzouni needs real harmonics to make viable the very TEA process that practicing scientists use to perform such discriminations.

5.2 Back to the first way: TEA ingredients for the Fourier harmonic

Before considering objections to the "second way" to an epistemic role, could the "first way" of ascertaining the epistemic artifacts of a Fourier harmonic succeed? Such an approach would involve verifying that all four TEA ingredients obtain between the observer and the Fourier harmonic itself. While I find it difficult to imagine what the epistemic artifacts of an infinitely-durative monochrome would be, one might suppose that an initial answer emerges from Azzouni's prior commitment to the reality of a behavior that the harmonic exhibits: superposition.

Specifically, Azzouni (2004a) endorses "[r]ecent experiments apparently illustrating thick epistemic access to superpositions of a particle . . ." (225, n. 3). The experiment referenced is that published by C. Monroe, D. M. Meekhof, B. E. King, and D. J. Wineland (1996),[44] which reports the manipulation and superposition of the quantum states of a Beryllium ion. I hypothesize that if these individual *quantum states* can be construed as (at least) thin posits, then because Fourier harmonics likewise superimpose into finite-duration pulses (thick posits referenced throughout science), then one may analogously claim TEA to *Fourier* superposition, and (at least) thin access to

[44] Thanks to Jody Azzouni (email correspondence) for this information.

Fourier harmonics. One flaw with this analogy, however, is that the quantum states in superposition are values of position and "angular momentum" (Monroe et al. 1996, 1132), the latter of which can be detected *independently of superposition experiments* (Halliday et al. 1997, 1030-1031),[45] and even "thick[ly]" in the deflationary nominalist sense (Bueno and French 2018, 176). Indeed, it could be said that angular momentum **grounds** the quantum superposition, as the Fourier harmonic **grounds** the optical superposition, but a harmonic in its infinite duration is not observed by itself. Thus, a side-by-side comparison of the TEA ingredients for quantum and optical superposition would remain fraught with disanalogy.

5.3 A Colyvanian approach, and the Coding Role

As I leave aside the orthodox or "first way" defense of the Fourier harmonic's epistemic role, I also decline the Colyvanian (2010) approach of claiming that the Fourier harmonic fulfills the Quinean virtues (which it does[46]), but that the excuse for not detecting harmonics in the raw is that they exist only as superimposed within the finite-duration pulses that we manipulate.[47] That proposal, despite appealing to Quinean virtues that Azzouni has abandoned as criteria for thin-posithood,[48] remains vulnerable to the "coding" objection of Azzouni and Bueno (2016),[49] an objection that equally threatens the (identical) excuse clause of my "second way" argument in section 5.1. The point of

[45] I refer to the "Einstein – de Haas Experiment," wherein a macroscopic iron cylinder rotates within a current-carrying solenoid.
[46] Michael Liston (2004; 1993) argues cogently for what amounts to a defense of the Quinean virtues of Fourier analysis.
[47] Note that this excuse clause is identical to that of the "second way" argument in section 5.1.
[48] For reasons outside the scope of this paper; see Azzouni (2012b).
[49] Discussed also by Azzouni (2009; 2004a, Chapters 8 and 9).

the coding objection is that indispensably mathematical sentences can be used "assertorically"—or in a way that commits the user to their truth[50]—without the mathematical terms referring, and that sentences of scientific theory can be asserted with the understanding that their "mathematical [ultrathin] posits are proxying for something empirical that we can't otherwise describe" (Azzouni 2004a, 173).

An example of a sentence with coding terms is, "The average star has 2.4 planets" (Azzouni 2009, 157). The sentence can be used assertorically, despite "average stars" and rational numbers not existing, Azzouni (2009, §5) argues, because of what he calls a "proxy norm" obtaining among scientific interlocutors. That is, because no one person can know all of science, the scientist uses "public" (152) sentences *D* assertorically for deduction and representation, with the implicit understanding that some of those sentences proxy for sentences *D** that other specialists could use assertorically (154). The point is not that *D** sentences could in principle always replace *D* proxies, but that *D** sentences enable one to draw appropriate implications from proxy sentences, including ontological implications (153-155).

As additional examples of proxy terms, Azzouni mentions "infinitesimals" and "the Dirac delta function" as "loosely-employed concepts" in science (154), concepts

[50] The "assertoric use" of sentences (which I sometimes call "assertion") is a tenet of Azzouni's deflationary account of truth. Azzouni (2009) sees assertoric use as a sort of converse of our linguistic practice with the Tarski biconditional. As a sentence like "'Snow is white' is true" can be shorn of its truth idiom and replaced with "Snow is white" (Azzouni 2004a, 16), so Azzouni (2009) calls it an "empirical fact" that when we "*assertorically use*" (141) some sentence W, we incur logical commitment to the sentence "'W' is true." Azzouni's point is that assertoric use transpires in scientific contexts and other deductions and descriptions, but not in a stage play, a quotation of another's words, or in various other contexts (Azzouni 2009, §2).

among which a deflationary nominalist might include the Fourier harmonic.[51] Similar to my claim that the infinite-duration harmonic is indispensable to reflectance ascription (section 4), moreover, so Azzouni and Bueno (2016) claim that scientifically recognized properties of "metal deform[ation]" depend indispensably upon "continua structural postulations" about real materials, a structure nevertheless "recognized [by scientists and some philosophers] . . . to be unreal" (794). Thus, against the supposition that Fourier harmonics exist hidden in superposition, the deflationary nominalist would likely call Fourier analysis a proxy language for whatever the electromagnetic field—or other presently obscure entity or process—is doing.[52] This objection appears to undermine the excuse clause both for the "Quinean virtue" argument of this section, and for my "second way" argument of section 5.1. If the harmonics indispensable to the stimulus and manifestation of SSR are just coding for something we cannot currently describe, then the harmonics are not thin posits, they are ultrathin.

5.4 Response to the coding objection

The coding objection to the thin-posithood of the Fourier harmonics that are indispensable to reflectance ascription appears sound and compelling,[53] because while it is one thing to claim that harmonics **ground** other posits and so possess an epistemic role (section 5.1), it is quite another to walk into a laboratory and assert that mirror A is "more reflective" than mirror B only if real mathematical entities (which possess infinite

[51] Although not shown in Figure 4.1, the frequency-domain representation of a single harmonic is a delta function with finite amplitude and unity bandwidth.
[52] This objection has a mechanical analogue: the "'third harmonic'" of a string does not vibrate, Liston (1993) alludes, *the string* vibrates (451).
[53] I answer later in this section whether the coding objection succeeds against the Colyvanian "Quinean virtue" argument of section 5.3, which, recall, does not involve reflectance ascription.

duration, by the way) are propagating through the room. I deny, however, that one can fairly, *universally* apply the coding objection (*viz*. the proxy norm) when identifying the very properties or entities by which **Grounding** obtains.

My denial hinges on an understanding of what kind of relation **Grounding** is supposed to be. At the end of the day, **Grounding** is "the [set of] detail-oriented scientific explanations (of how *this* specific property of *that* enables us to track it because of certain causal interventions we're consequently capable of) . . ." (Azzouni 2004a, 134). There is a lot to unpack in the previous sentence, and I cannot elaborate all of it in the remaining space of this paper, but the overriding point is that **Grounding** is an explanation, and I have not seen Azzouni endorse a specific account of explanation that either supports or undermines the notion that mathematical entities explain physical phenomena such as observation, the data accumulated through TEA, or the obtaining of TEA conditions like **grounding**.[54]

Indeed, Azzouni (1998, 12) simply punts on the question: "explanation operates at the sentential level, and is indifferent to how we tease out the ontological commitments of the sentences which provide the explanations we take seriously." Hence in declining to give an account of extra-mathematical explanation proper,[55] but freely allowing the proxy norm to quash the reification of mathematical posits indispensable to the science of continuum-bent metals (section 5.3), Azzouni appears to *assume* that mathematical posits are never going to incur an epistemic role, and so never play more than a representative

[54] For an introduction to the ongoing controversy over whether mathematics can explain physical phenomena, see Marcus (2015, Chap. 7).
[55] "Extra-mathematical explanation is the . . . mathematical explanation of physical facts" (Baker and Colyvan 2011, 326).

or descriptive role in scientific explanations (the set of which includes **Grounding**). He says as much when he remarks in passing that mathematical entities do not explain physical phenomena, because mathematical entities do not exist (Azzouni 2012, 964). But when the *obtaining* of a **Grounding** explanation depends on a mathematical property like infinitude (as Fourier harmonics **ground** human-visual TEA to vases by making reflectance ascribable), Azzouni's unargued dismissal of mathematics as non-explanatory begs the question, by deflationary nominalism's own lights.

Table 4.1: A question-begging application of the proxy norm

Can a mathematical posit be thin?	⟶	Consult the TEA conditions, to see if the posit possesses an epistemic role.	⟶	Fourier harmonics indispensable to the obtaining of the TEA condition of **Grounding** possess an epistemic role (section 5.1).	⟶	Then render those mathematical posits ultrathin by the proxy norm (ignoring TEA conditions). Why? Because mathematical posits do not exist (*petitio principii*).

Table 4.1, albeit free of "explanation" language (which I provide below), outlines how I think that appeals to the proxy norm can beg the question about the thin-posithood of a mathematical entity. Simply put, consulting and ignoring TEA conditions in the same argument, on the same question, is invalid. An alternative rendering of the same point is that categorically denying that mathematical entities could explain physical phenomena (Azzouni 2012, 964), but then appealing to the *proxy norm* to preserve that denial when mathematics proves indispensable to the ascription of properties within a **grounding** explanation of physical phenomena, is to deny without argument that mathematical entities explain physical phenomena. The proxy norm, in other words, is

not an argument that mathematical posits do not exist, but an implicit assumption that the TEA conditions successfully preclude the existence of mathematical posits. My reflectance counterexample challenges that assumption, and doubling-down on an assumption (the proxy norm) against a counterexample is begging the question.[56]

Granted, by the phrase, "when mathematics proves indispensable to the ascription of properties," in the previous paragraph, I mean indispensable to blocking conceptual regresses like those identified in section 4, and I assume the metaphysically serious (*viz*. realist) meaning of property "ascription" articulated in footnote 41. Although I lack space to develop the distinction formally in this paper, I think it is these conceptual coherence problems with property ascription, rather than the apparently commoner problems of tractability or the accuracy and precision of property values, that truly disrupt an "explanation" in the generic and unspecified sense (of the deflationary nominalist), and that render a given explanation meaningless.

For example, I find it entirely disingenuous and meaningless to ascribe (e.g.) pulse-reflectance to a surface, when one knows that "pulse-reflectance" is a contradiction in terms like "non-cubical cube" (section 4.1). On the contrary, it seems neither meaningless nor disingenuous to ascribe some physical property with value π, and then say that the property's value is approximately 22/7. (Indeed, this looks like a simple

[56] A weaker defense against the proxy norm objection to Fourier **grounding** is that Fourier **grounding** could remain an *explanation*, even if scientists did not accept that explanation, just as shadows of length m explain flagpoles of height n according to primitive forms of deductive-nomological (DN) explanation, even if few or no theorists accept that particular explanation or explanation-form, and even if all flagpoles were incinerated in a nuclear disaster that blotted out the sun (eliminating all shadows). It is probably not to my dialectical advantage, however, to analogize Fourier **grounding** to bad DN-explanations, and I think that the stronger charge of question-begging, in the main text above, holds.

exchange of *D*- and *D**-sentences.) Nor do I disagree with Azzouni (2004a) that assertions of Newton's law, "**F**=*m***a**," sometimes "*codify* the differential equations governing" the motions of masses, rendering that motion tractable for discussion (182-183). Forces and π-valued properties do not lose their *meaning* or become patently self-contradictory through these shortcuts, as reflectance does when one tries to approximate away or replace its harmonic constituents. Saying that "π is 22/7" just seems like a different and more acceptable kind of assertion than "π is a non-cubical cube."

Hence property ascriptions that do not suffer conceptual regress, in my view, remain *rightly susceptible* to the proxy norm, even when those properties possess indispensably mathematical definitions and appear in **grounding** explanations. The continuum-bent metal discussed by Azzouni and Bueno (2016, 793-794) and mentioned in section 5.3, for example, might feature indispensably in a **grounding** explanation,[57] but that featuring alone does not suffice to block the proxy norm, since eliminating the continuum as unreal might result only in intractability or inaccuracy about a physical property's value, and not result in the regressive destruction of that property's very concept and ascribability, as nominalizing a harmonic destroys reflectance (sections 4-5.1).[58] Of course, marking this distinction between conceptual coherence and tractability or accuracy is not to *argue* for it, but I present it to avoid begging questions in the other direction: that every mathematical posit referenced in a **grounding** explanation is thin,

[57] As might the continuum-divisible space analyzed by Azzouni (2004a, Chapters 8 and 9), and the continuum-defined "background geometry" of string theory (Azzouni 2009, 161).

[58] If I am wrong in this assumption, and mathematically nominalized properties of continuum-bent metal suffer a conceptual regress analogous to the VRR, then that fact only supports my argument.

because no formal deflationary nominalist account of explanation nullifies that thin status. I suspect that the properties ascribed in **Grounding** explanations cannot be ascribed as "non-cubical cubes" would be ascribed, and that this analogy serves as a preliminary heuristic for modulating the proxy norm within **Grounding**.

To tie up one loose end, then, I also find the proxy norm rightly applied against the free-standing Fourier harmonics alleged to exist in undetectable zero-sum superposition in the Colyvanian "Quinean virtue" argument of section 5.3. That example does not obviously involve **grounding**, nor does the superposition in question achieve any obvious traction with other TEA conditions to mark out the harmonic's epistemic role (section 5.2).

Lastly, it bears mentioning that Azzouni's (2017; 2012a) antirealism about properties does not clearly alter any of the aforementioned problems or arguments. The word "properties" still features prominently in the TEA conditions, and Azzouni (2017, Chap. 8) still endorses TEA without listing revised or "property-free" TEA conditions. Even if there is nothing on the surface of a vase above and beyond interactions of "per wavelength" electromagnetic propagations (and even if, as Azzouni (2017, Chap. 6) attests, there are no real *surfaces* in the universe), an account of the optical interactions that explain our observations and the vase's existence may incur more difficulty than some are likely to anticipate, if we do not appeal to real Fourier harmonics.[59,60]

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[59] For the record, optical properties besides reflectance appear to suffer a conceptual regress analogous to the VRR. Examples include the dispositional refractivity that Chakravartty (2007, Chapters 2-3) implicitly endorses, and the surface plasmon resonance ascribed by Bursten (2018).

[60] Suitable for footnote-length mention here is how unhelpful it would be to call Fourier harmonics an "idealization" in the deflationary nominalist sense. Azzouni (2005) refers to idealizations as "falsifications" (34), and as a process of "systematically excluding

6. Conclusion

My goal in this paper has been to identify an epistemic role for some mathematical entities, not by satisfying all four TEA conditions with respect to those entities, but by showing them indispensable to the ascription of properties that facilitate TEA to "thick" posits, and by arguing that the mathematical entities perform a **grounding** role in such TEA (section 5.1). I identified this **grounding** and thus epistemic role for the Fourier harmonic by arguing that its infinite duration is indispensable to blocking a conceptual regress of the reflectance property that Azzouni (2010, 30) ostensibly ascribes to vases. I then argued that appealing to the deflationary nominalist "coding" objection of Azzouni and Bueno (2016), to discount the harmonic term as a non-referring proxy for some to-be-had non-mathematical theory about propagating light, amounts to a *petitio principii* within the **Grounding** context (section 5.4). One target for further research is a scientifically-informed metaphysical account of what it means for properties to be sufficiently "ascribed" to perform their **grounding** role(s) in deflationary nominalism. Such an account could alter my present conclusion, which is that the Fourier harmonic exists as a thin posit.

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phenomena (such as friction) from explicit consideration to make derivations tractable" (30, n. 16). As I have argued in section 5.4, I do not have a tractability problem with reflectance, but a conceptual coherence problem. Thus, idealizing harmonics would not rescue reflectance ascription, and the **Grounding** condition between the viewer and the vase would still be obliterated, an unwanted result for the deflationary nominalist.

CHAPTER 5

AN EXTRA-MATHEMATICAL PROGRAM EXPLANATION OF COLOR EXPERIENCE*

1. Introduction

In the debate over whether mathematical facts, properties, or entities[1] could explain physical phenomena, Aidan Lyon's (2012) affirmative answer stands out for its employment of the "program explanation" (PE) methodology of Frank Jackson and Philip Pettit (1990). Program explanation is a form of ontic (Saatsi 2016, 1046) or "constitutive explanation" (Pettit 1993, 69), whereby real properties in the world serve as explanantia for a physical event that is the explanandum.[2] Essential to PE theory, moreover, is that programming properties explain *without causing* the explanandum. This feature of PE undergirds some explanations from mental properties ("attitudinal contents") and sociological properties ("group-cohesion") (Jackson and Pettit 1990, 115),[3] and Lyon adds to these applications the primeness of integers 13 and 17 program-explaining the North American *Magicicada's* 13- or 17-year dormancy period.

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*Submitted to *International Studies in the Philosophy of Science*, on February 02, 2021.

[1] I treat these three terms interchangeably, context permitting.

[2] Some may object that PE can be construed as an epistemic explanation that advances human understanding rather than picking out worldly ontology (thanks to a participant at *Ernst Mach Workshop IX: Non-Causal Explanations in Physics*, September 18, 2020, for this point), but to engage that controversy in this paper would be a digression.

[3] More recently, Shea (2018, §8.3) endorses program explanations of human behavior.

The cicada explanandum-question, more precisely, is why cicada nymphs live in the ground for 13 or 17 years before emerging, mating, and dying, given ecological conditions that constrain dormancy periods to 12 to 18 years.[4] Lyon argues that avoiding periodic co-emergence with the life cycles of predators proves evolutionarily advantageous to the cicadas, and that prime periods of cicada emergence minimize such intersections. Hence Lyon sees an "extra-mathematical"[5] explanation to obtain between the primeness property of number theory and the actual dormancy periods of cicadas. Primeness "programs" the periods in a sense to be articulated in section 2, just as other mathematical properties program patterns and events in additional examples of mathematical indispensability, including Plateau's laws for soap films, the Bridges of Königsberg, the hexagonality of honeycomb, etc. (see Lyon 2012).

More than merely explain events via mathematical properties, however, Lyon (2012) reifies explanatory mathematics, running his PE conclusion as a premise in the Enhanced Indispensability Argument (EIA):[6]

> [B1] We ought to be committed to the existence of all and only the entities that are explanatorily indispensable to our best scientific theories.
>
> [B2] Mathematical entities are explanatorily indispensable to our best scientific theories.

The conclusion being that mathematical entities—the integers 13 and 17 in the cicada case—exist (Lyon 2012, 573). Lyon understands his substitution of PE for premise B2 in

[4] For biological and ecological details, see Lyon (2012, 561, 567, n. 10), Baker (2005, §2), or Berenstain (2017, §3.4).

[5] "Extra-mathematical explanation is the . . . mathematical explanation of physical facts" (Baker and Colyvan 2011, 326).

[6] The following text of B1 and B2 is Lyon's (2012, 572) paraphrase of Baker (2009), in my formatting.

the EIA to advance the indispensability debate in favor of mathematical realism, since PE secures for mathematics an explanatory versus a merely descriptive role in our best scientific explanations (572).

While I sympathize with Lyon's two-part argument (Part 1: extra-mathematical program explanation; Part 2: EIA), the first part faces powerful objections by Juha Saatsi (2012; 2016). In this paper, I argue that Saatsi's objections fail against an extra-mathematical program explanation that Lyon and other indispensabilists have overlooked: Frank Jackson's (1998a) program explanation for the human experience of seeing surface colors. In addition to answering Saatsi's "Part 1" criticisms, however, my analysis of Jackson's counterexample also appears timely for "Part 2" EIA concerns, since Saatsi has recently declared debate on the EIA (of which program explanation is an eligible premise) to have "reached a serious impasse" (Knowles and Saatsi 2019, §1). Circumventing this impasse, I argue that program explanation reifies some mathematical entities *without* the EIA, specifically the mathematical entities that I find indispensable to Jackson's PE for color experience; those entities are Fourier harmonics, infinite-duration monochromatic sinusoids. Fourier harmonics must be real, I will argue, if the dispositional property of "reflectance" central to Jackson's PE is to manifest in a conceptually coherent sense.[7]

Two conclusions follow, then, from my identification and defense of an extra-mathematical program explanation for color experience (hereafter EMPEC). The first is that Saatsi's criticisms of extra-mathematical PE are not universally applicable and

[7] I identified this conceptual coherence problem for reflectance, and its Fourier solution, in Danne (2020).

decisive; EMPEC stands as a counterexample to them. The second conclusion is that some program explanations reify the mathematics indispensable to PE explanantia without the EIA, since the *property* realism inherent to program explanation entails a limited mathematical realism in those cases. The contingency of a limited mathematical realism on property realism has been underacknowledged by indispensabilists, and I use program explanation to elucidate that contingency.

My roadmap for this paper is to review program explanation in section 2, and Jackson's PE in section 3. I then motivate the mathematization of Jackson's PE in section 4, arguing along the way that Jackson's reflectance realism reifies harmonics without the EIA, and I argue in section 5 that Saatsi's objections to extra-mathematical PEs fail against EMPEC. Section 6 concludes.

2. The Brass Tacks of Program Explanation

Helpful to understanding program explanation are Jackson and Pettit's (1990, 108) criteria for non-causal programming properties (I omit criteria 1 and 2 for brevity):

> 3. A property F is not causally efficacious in the production of an effect *e* if these three conditions are fulfilled together.
>   (i) there is a distinct property G such that F is efficacious in the production of *e* only if G is efficacious in its production;
>   (ii) the F-instance does not help to produce the G-instance in the sense in which the G-instance, if G is efficacious, helps to produce *e*; they are not sequential causal factors;
>   (iii) the F-instance does not combine with the G-instance, directly or via further effects, to help in the same sense to produce *e* (nor of course, vice versa): they are not coordinate causal factors.

An example of a programming property (F) is the "temperature" of boiling water; *ex hypothesi*, temperature does not cause the cracking of a sealed glass container of boiling water, the momentum (G) of such-and-such a water molecule does (110). Temperature neither 'combines' with nor 'precedes' molecular momentum to crack the glass, but temperature *programs* the cracking by making "probable" that some-molecule-or-other will have the momentum to crack the glass (114).[8] Hence an epistemic advantage accrues to the program-explanationist, Jackson and Pettit (1990, 117) claim, over that of the process-explanationist who studies (G) causes alone. The theorist who knows that the water was boiling possesses a better explanation, with stronger modal information, than the process-explanationist who studies only glass-bonds and molecular velocities.

In a similar vein, Lyon (2012) claims that the primeness (F) of integers 13 and 17 "ensures the instantiation of a causally efficacious property" (566) or sequence of properties (G), by which the cicadas consistently settle on a 13- or 17-year dormancy period. Primeness neither 'combines' causally with the cicadas' environmental stressors, nor does primeness causally 'precede' those stressors. Primeness rather ensures or makes probable the existence of a *minimum* period of lifecycle intersection between cicadas and their predators, given an environment and its resources. Had the ecological history of cicadas been different than it actually was, for example, their lifecycle periods would probably still be prime, or become prime (567-568).

An additional point, which will be important later, is that Jackson and Pettit (1990) allow program explanations which *lack* modally strong information (relative to G-

[8] The probability-raising function of programming properties, which Jackson and Pettit (1990) sometimes describe with the verb "ensures" (114), has been a controversial tenet of program explanation. Thalos (1998, 286-289ff) criticizes the notion that a probability-raising factor in an explanation could be non-causal, and Macdonald and Macdonald (2007, §3) reach a similar conclusion. I respond to Saatsi's (2012; 2016) objections to the "ensuring" relation in section 5, but cannot otherwise assess the merits of PE as a kind of explanation in the scope of this paper.

processes) to nevertheless *be* program explanations (116).[9] One example is knowing that a vase's microstructure which cracks-like-so (G) is also "fragile" (F); fragility trivially programs the microstructure, because fragility ensures without precession or combination that the microstructure cracks-like-so (G), but knowledge of fragility (F) yields no *better* (modal) explanation of the cracking than does knowledge of (G) (116). I call these uninformative program explanations "inert" or "impoverished,"[10] and I refer to them in my discussion of color experience PEs.

3. Programming Color Experience

The event in the world that Jackson (1998a) seeks to explain is the human experience of seeing color, and the disposition that he finds to program this experience is the disposition of objects to look (e.g.) red to normal observers in standard conditions.[11] Important to immediately distinguish, then, is the *perceptual* disposition of objects to look colored, from the *reflectance* disposition—"surface spectral reflectance" or SSR—to which some color objectivists ontologically reduce color (e.g. Byrne and Hilbert 2003, Hilbert 1987). Jackson (1998b, 87) affirms the reality of both dispositions, but he rejects the ontological reduction of color to SSR, objecting that the reduction does not go far

[9] Jackson and Pettit (1990, 116): "A program explanation of an event *may* provide information which the corresponding process explanation does not" (emphasis mine). I take this "may" qualifier to mean that program explanations do not necessarily provide the sought-after modally-superior information.

[10] Bliss and Fernández (2010) give an accessible and convincing heuristic for distinguishing impoverished from non-impoverished PEs, but because those authors' conclusions sometimes contradict Jackson and Pettit (1990, and elsewhere) about particular cases, I decline to elaborate Bliss and Fernández's account, to avoid internal debates about PE proper.

[11] These conditions are spelled out by Hardin (1988, Chap. 2). One may note that by making objects the bearers of the disposition to look red, Jackson implicitly rules out various subjectivist ontologies of color, according to which color experience might be programmed or caused by observers, brains, or minds, etc.

enough: color is (ontologically) instead the disjunctive, microstructural, categorical *base* of SSR rather than SSR *qua* disposition (110).[12]

One reason that Jackson denies the reduction of color to SSR, inferable from section 2, is that he understands dispositions to be non-causal higher-order[13] properties (F) of entities that possess causal, categorical base properties (G); he would find it "ontologically extravagant" to attribute causality to an impoverished higher-order property like fragility or SSR (Jackson 1996, 202). Once one knows that a given microstructure causes color experience, nothing is gained by learning that the surface composed of that microstructure is also "reflective."

Secondly, Jackson (1996) takes dispositions to be non-causal by reason of the mutability of physical laws across possible worlds. Some "poisonous" categorical microstructure might not kill humans in worlds with different "laws of nature," Jackson claims, but were poisonous*ness* a causal disposition, it would remain "intimately connected" to swallowing and human death "in every world" (203). Hence Jackson denies that "properties . . . have [their] causal powers essentially" (203) or independently of laws, but he interprets any dispositions lacking a categorical base *to* possess their causal powers essentially, with manifestations that transpire "uncaused" (204). Thus, dispositional SSR cannot *be* color, in Jackson's (1998a) account, because color should causally explain human color experience (86).[14]

[12] Hilbert (1987, 2089-2092) and Byrne and Hilbert (2003, 53; cf. 20, n. 25) deny that color reduces ontologically to microstructure, but this point does not affect my argument, nor need I take sides. On the distinction between categorical and dispositional properties, see Schrenk (2017, Chap. 2).

[13] An object's higher-order property is the property-of-having a lower-order property; thus "fragile" entities have the property-of-having a microstructure that shatters-like-so.

[14] See McFarland and Miller (1998; 2000) for criticism.

From Jackson's (1998a) program explanation standpoint, then, the perceptual disposition to look red is a real property that programs the disjunctive microstructural base that is redness, and this disjunctive base, *qua* disjunctive, programs a particular event of looking red in addition to causing it (87). The disjunctive base increases the probability that *some* such base causes *this* experience, but the disjunctive set furthermore straightforwardly causes the event, as the "depth" of a wound within some range causes death (87).[15]

Enter my insight about dispositional SSR. What Jackson (1998a) omits to mention is that the categorical base of the disposition to look red *could in principle be* the categorical base of dispositional SSR; for microstructure is microstructure, and it is an empirical question whether the correlations that Hilbert (1987) and other color objectivists tender between human color reports and SSR values further correlate to or circumscribe a nameable set of microstructures.[16] Hence just as Jackson and Pettit (1990, 115) allow programmatic chains through multiple non-causal properties to a terminating causal property (F1→F2→G), one may ask why the disposition to look red (D_R) could not program SSR, which would in turn program the disjunctive causal base (G) of both dispositions: (D_R→SSR→G). Nor would this maneuver automatically "impoverish" the PE for color experience by introducing a higher-order disposition (SSR) that Jackson considers inert. That causal bases (G) are "reflective," and that a surface with an SSR

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[15] See McFarland and Miller (1998; 2000) and Wright (2003), for criticism.

[16] I remind the reader that I am not defending color objectivism or providing any reason to believe it. The view is controversial, as Gert (2017, Chap. 3) recently argues on scientific and philosophical grounds.

profile[17] has the disposition to look colored, may both be trivial claims; but that a given surface looks red (D_R) still makes probable (*ex hypothesi*) that some disjunct-or-other in the (G)-array causes red experience.

Jackson's (1998b) *implicit* doubt about my (D_R→SSR→G) programming chain[18] is that SSR correlations to color experience, and microstructural (G) correlations to color experience, could anti-correlate with respect to their concepts (111). That is, while the reflectance profiles of red objects should closely resemble the profiles of pink objects (neither of which should resemble the profiles of green or blue objects), "[t]here is no reason to think the physical property we are latching onto when some particular thing looks red is similar to that we are latching onto when some particular thing looks pink . . ." (111). Jackson suspects that colors causing similar experiences (two slightly distinguishable hues of red) could lack "distal commonalities" in their causal bases to such an extent that "[r]edness" would no longer be "the property in common to red things," and color eliminativism or a perceptual error theory about color would result (112). Thus, as I understand Jackson's argument, he would rather ignore SSR correlations and assume that color is a "not excessively disjunctive" set of (G) bases (Jackson 1998b, 108).

In response, I am not sure that we should demarcate reasonably heterogeneous disjunctions of causes by *absconding* from measurement technologies like the spectrophotometry that measures SSR. To learn very much about microstructures at all,

[17] An SSR profile is the set of a given surface's reflectance values across the human visible spectrum (400-800 nm). Hilbert (1987, Chap. 4) sums and integrates some of these values for his color reduction, but the details do not affect my argument.

[18] I say "implicit" because Jackson (1998b) does not explicitly mention programming.

they must be reflective in *some* sense; even an atom needs to respond to electromagnetic stimulus somehow, if quantum-physical (and even classical-physical) claims are to be tested.[19] Scientists and philosophers universally define the reflectance needed to study (G) structures as a "per wavelength" efficiency to absorb and reemit light, and in section 4, I generate a conceptual regress about this received view that motivates my mathematization of SSR and extra-mathematization of Jackson's PE. In the meantime, the point is that Jackson's PE necessarily incorporates the property of reflectance (SSR) as an explanans, because the claim that base properties (G) cause color experience *without* reflecting light is implausible.

Taking stock, I have argued that (D_R→SSR→G) is an entirely reasonable version of Jackson's PE for color experience.[20] As I have argued in Danne (2020), however, there is a conceptual regress about SSR (equally applicable to its base G) that renders it unfit for ascribing as a property of anything. Unfit, because self-contradictory: the received definition of SSR turns out to mean something analogous to "non-cubical cube," and I deny that such a conceptually incoherent property could cause or explain color experience. The swiftest way to block this regress is to mathematically redefine SSR, which I do in the next section; this redefinition, in turn, renders Jackson's PE for color experience extra-mathematical, transforming the (D_R→SSR→G) chain, with some modifications, into EMPEC.

[19] I thank Bob Mullen for an insight along these lines.

[20] Jackson (1998b, 87) all but affirms the reasonableness of this PE chain when he says: "We know that objects have dispositions to look one or another colour [D_R], *that they have dispositions to modify incident and transmitted light in ways that underlie their dispositions to look one or another colour* [SSR], [and] that they have physical properties [G] that are responsible for both these dispositions . . ." (emphasis and brackets mine).

4. Mathematizing and Ascribing Reflectance

4.1 The Vicious Reflectance Regress

As I argue in Danne (2020), the main problem with ascribing SSR is that its most accessible and familiar definition is an operational definition that launches a regress of the reflectance value at any given wavelength, leaving the SSR concept incoherent and not ascribable as a *real* PE explanans.[21] Granted, the wording of SSR's received definition appears non-operationalist, suggesting instead dispositional realism:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987, 1037-1041)

In this quote, philosopher of perception David R. Hilbert calls SSR a "stable property of objects," ostensibly denying that SSR is an operational property whose value depends on measurement method; but operational (and regressive) the definition is, nevertheless.

The reason why, is that "flux" in Hilbert's definition equates to average power in watts (Hilbert 1987, 1033-1042), but "average power" is itself an operational colloquialism unsuitable for the metaphysics of property ascription. According to orthodox signal theory, "average power" is technically only computable for infinite-duration signals, since signals with average power possess infinite energy; finite-duration signals with finite energy possess zero average power (Haykin and Van Veen 1999, 20-21). But why do orthodox signal theorists adopt this convention, and why should

[21] I do not deny that operational properties (like temperature defined as "what the thermometer says") could serve as PE explanantia, but I deny that regressive or conceptually incoherent properties can serve as PE explanantia.

philosophers care? The answer to both questions is that the orthodox convention *prevents* the very conceptual regress at issue in SSR (and other electromagnetic) ascriptions, about how much of a signal's energy or power is propagating "per wavelength," a consideration apropos to ontological color reductions.

To be clear, calculating a shorthand "average power" for finite-duration signals (i.e., "pulses")[22] proceeds effectively in science and engineering, but only because scientists are not ascribing *properties* in any robust sense (if I am wrong, then what are the International System (SI) units for ascription, instantiation, dispositionality, categoricity, intrinsicality, universals, tropes, etc.?). The philosopher Hilbert (1987) does ascribe reflectance using the laboratory shorthand, but I will argue in this section that doing so generates an asymptotic conceptual regress of the reflectance value, at any given wavelength, to the meaningless and undefined value of 0/0.[23] I deny that any property possessing this value could obtain at a surface, much less explain color experience.

The asymptotic plunge to 0/0 of any given pulse-SSR value I call the Vicious Reflectance Regress (VRR), and it launches as soon as one considers an empirical phenomenon that afflicts every finite-duration pulse. That phenomenon I call "harmonic dispersion," the inverse relationship of a pulse's duration to its bandwidth.

[22] By "pulse," I mean a finite-duration sinusoid like Figure 5.1 (a) below, although my arguments of this paper apply to other-shaped pulses used by laboratories, such as Gaussian and hyperbolic-secant.

[23] Recall from Hilbert's block quote above that reflectance is a ratio of average powers. Hilbert commits a slight miswording in that quote, suggesting that "incident" flux occupies the ratio's numerator. This allusion is inconsistent with the rest of Hilbert (1987, Chap. 3), which puts reflected flux in the numerator and incident flux in the denominator; cf. Jackson (1998b, 109), who correctly paraphrases Hilbert with reflected flux in the numerator.

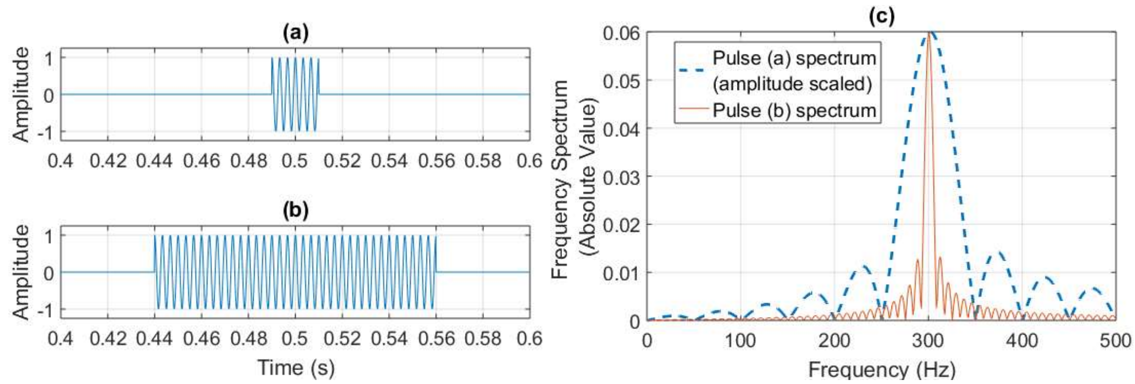


Figure 5.1: Harmonic Dispersion [6x duration]

Figure 5.1 depicts harmonic dispersion for pulses with sub-optical carrier or center frequencies (for ease of modeling).[24] Pulse (b), whose duration is six times that of pulse (a), exhibits the greater monochromaticity of the two in graph (c), which depicts the "per wavelength"[25] power of the longer pulse (solid line) as more tightly concentrated about center frequency 300 Hz than is the power of the shorter pulse (dotted line). The upshot of Figure 5.1, I will argue, is that pulse-SSR is a contradiction in terms like "non-cubical cube," because pulses do not propagate "per wavelength"; a pulse instead propagates as an envelope of wavelengths (each trace of Figure 1 (c) is an "envelope"), and any finite-duration per-wavelength components of these envelopes are themselves envelopes, *ad infinitum*. Thus, *contra* Hilbert (1987), there *is* no *pulse* power which dissipates at (e.g.) 650 nm; something besides a "pulse" dissipates at 650 nm (see below, and section 4.3).

[24] Visible light propagates in the THz (10^{12} Hz) range, but harmonic dispersion occurs at all usable frequencies and pulse durations, including those of radio and radar.

[25] "Frequency" and "wavelength" are interchangeable in physical optics. Frequency is speed of light divided by wavelength.

I launch the VRR by assuming that an optical pulse centered at 650 nm,[26] with average power 5 Watts (W) and sub-picosecond[27] duration, propagates toward a perfect mirror with SSR = 1 across the human-visible band (400-800 nm). Laser researchers report such pulses to exhibit harmonic dispersion as wide as 100 nm (Stingl et al. 1995; Deng et al. 2005), so we can assume that our 650 nm pulse dissipates appreciable power up to 700 nm, and down to 600 nm, analogous to the 225 Hz and 375 Hz side-peak dissipations of the dotted-line spectrum in Figure 1 (c). For calculational convenience, I assume that 20% of the 650 nm pulse dissipates at non-650 nm wavelengths (the accuracy of this estimate turns out to matter little, because a regress is a regress).

The crucial, and apparently easy question at this point is: "How many watts will reflect from the perfect mirror at 650 nm?" According to the parameters I have laid down, the answer seems to be 80% of 5 W = 4 W. But the answer cannot be 4 W, I contend, because that 4 W, 650 nm component of the original pulse *itself* possesses a sub-picosecond duration (what other duration could it have?). Hence by the empirical law of harmonic dispersion, that 650 nm, 4 W "component" must itself again disperse energy to its sideband wavelengths and become only 3.2 W, a pulse of short duration which must re-disperse, *ad infinitum*.

[26] As the pulses of Figure 1 are centered at 300 Hz.
[27] 1 ps = 10^{-12} seconds.

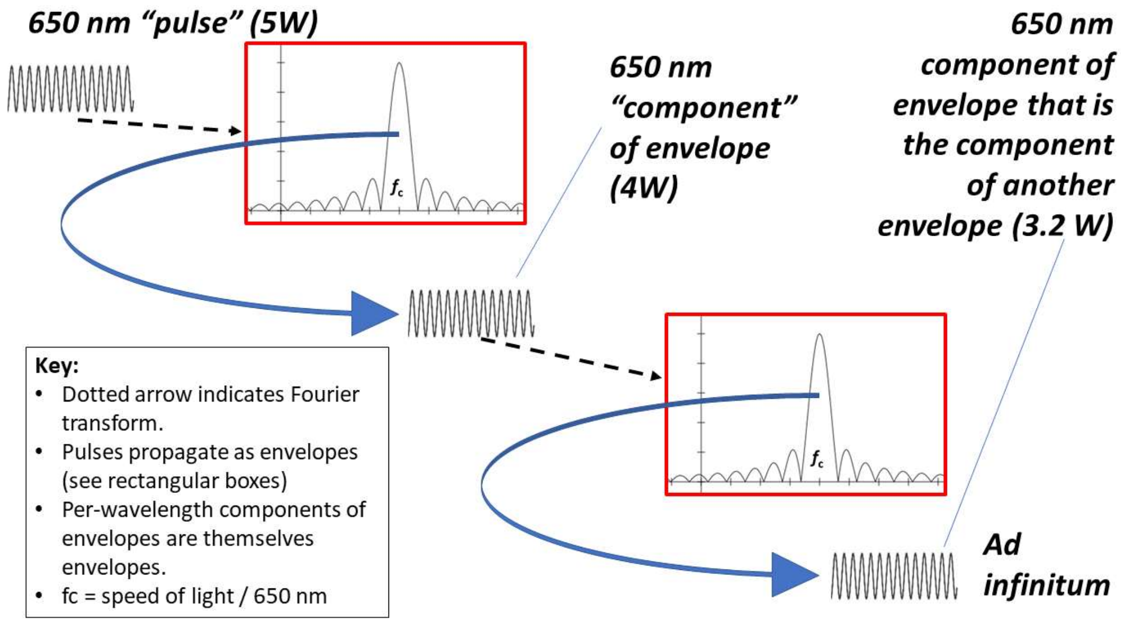


Figure 5.2: The Vicious Reflectance Regress

The 650 nm sinusoidal "component" of the original pulse in Figure 5.2 loses power with every iteration, falling asymptotically to zero. Important to note, moreover, is that I have not shown this regress to be a *physical* one, and I do not need to. The scientist likely measures 4 W at 650 nm, and thinks nothing further of it; but whatever the scientist measures is not a *pulse*, by the argument just given. The pulse-SSR value at 650 nm asymptotically approaches 0/0, which is nonsense considering that the incident pulse is 5 W, dispersion is only 20%, and the mirror is assumed perfectly efficient. Due to this conceptual incoherence, pulse-SSR is not sufficiently well defined to be a real property. Jackson's PE for color experience runs aground, then, because the categorical base of pulse-SSR is only usefully described along a "per-wavelength" dimension,[28] but the VRR militates against the propagation of finite-duration light in that dimension.[29]

4.2 Two Intermediate Objections

Before proposing my mathematical re-definition of SSR that blocks the VRR, two objections merit dispatching.[30] The first is that I too severely demand completeness in the definition of a respectable scientific property (reflectance); the second is that the VRR has nothing to do with *color* objectivism, or the ontological *color reduction* that interests Jackson (1998a), since I run the VRR at picosecond durations in which humans cannot distinguish colors (Scase and Foster 1988). Per the first objection, one could claim that other real properties, like the mean kinetic energy (MKE) definition of temperature, is scale-bounded in its definition just like SSR appears to be (since harmonic dispersion is negligibly small for long-duration signals). Peter Smith (1998), for example, argues that "*there is no fact of the matter*" (39) about the value of a substrate's MKE temperature beyond a few decimal places (40-41), since particle velocity through the vanishing sphere used to estimate kinetic energy becomes undefined. I reply that the MKE problem is one of chasing asymptotic *accuracy*, unlike the VRR which destroys the reflectance value on both sides of the decimal place (by sending the reflectance value to 0/0). MKE temperature values possess meaning when asserted to only so-much precision, but pulse-SSR loses its meaning entirely, no matter what dispersion percentage is assumed. A 0.001% regress, for example, is just as vicious as the 20% regress assumed in section 4.1,

[28] See any text on physical optics, such as Born and Wolf (1999).

[29] Appealing to the particle-wave duality of light does not block the VRR, because photon absorption and reemission also occur "per wavelength" in finite time.

[30] See additional objections to the VRR, with replies, in Danne (2020).

because both percentages iteratively plunge the per-wavelength pulse power to zero (Figure 5.2).

Secondly, and for the same reason, it does no good to object that humans fail to discriminate colors in the sub-picosecond range, because the object of my investigation is the metaphysics of program explanation rather than perceptual theory. I have argued that pulse-SSR is too regressive and incoherent to ascribe as a *real* program explanans; and Jackson (1998a) does not treat any property in the color experience PE as operational; he posits real dispositions and bases responsive to natural laws in the actual world. Harmonic dispersion is undeniably one of those laws, and so the program explanationist needs a better-defined reflectance property than pulse-SSR. I propose such a redefinition next.

4.3 The indispensability of Fourier harmonics to reflectance ascription, and their reality

The most straightforward way to block the VRR is to heed the orthodox signal-theoretic meaning of "average power." In this vein, I redefine reflectance from the per-wavelength disposition to reflect "pulses" (section 4.1), to the per-wavelength disposition to reflect the (optical) Fourier harmonics that superimpose without remainder into pulses; for due to their infinite duration, harmonics never disperse, and so never launch the VRR. I have already illustrated Fourier harmonic superposition, moreover, in Figure 5.1. Every plotted point in Figure 5.1 (c) indicates the amplitude of an infinite-duration Fourier harmonic oscillating at the single frequency listed on the horizontal axis. Superimposing all such infinite-duration harmonics yields the finite-duration pulses shown in Figure 5.1 (a) and (b) (by the Fourier transform).[31,32]

Thus, when the laser scientist measures a "4 W" reflection at 650 nm, in my account (see section 4.1), she is measuring some portion of an infinite-duration optical *harmonic* at 650 nm, a portion not cancelled-out by its neighboring harmonics.[33] No matter the duration of the incident "pulse," the reflective efficiency at 650 nm is "stable" and unchanging, because only a constituent harmonic of a pulse, and not the pulse simply construed, is reflecting at 650 nm; the VRR is vanquished.

The appropriate program explanation for color experience, therefore, becomes ($D_R \rightarrow SSR_H \rightarrow G$), where SSR_H is the disposition of a surface to reflect optical harmonics instead of pulses. This programming chain is EMPEC, and interestingly, it reifies Fourier harmonics *without* the EIA, since SSR_H—if it is to be stimulated and manifest at all—must do so via *real* infinite-duration waves (harmonics), on pain of launching the VRR (sections 4.1-4.2).[34] This EIA-free mathematical realism contingent on property realism,

[31] A per-wavelength phase shift is also needed for the Fourier transform, along with negative-frequency harmonics, depictions for both of which I omit for clarity.

[32] As a result of this Fourier harmonic redefinition of reflectance, one can see the intentional error that I exhibited in Figure 5.2 for didactical purposes, about ascribing electromagnetic properties with finite duration signals (the Hilbert 1987 method). What I drew as finite-duration sinusoidal oscillations in that diagram (extracted from the envelope) actually need to be infinite-duration oscillations, to end the regress with the 650 nm oscillation possessing a stable 4 W of average power.

[33] Akin to how the portion of the pulse between 0.49 and 0.51 seconds in Figure 5.1 (a) is not cancelled-out by the dotted-trace harmonics in Figure 5.1 (c), but the regions of Figure 5.1 (a) outside 0.49 and 0.51 seconds *are* cancelled out by that superposition.

[34] I am admittedly being vague about what kind of mathematical realism follows from the reality of harmonic-SSR (SSR_H). Are the harmonics real as Aristotelian immanent universals, transcendent Platonic universals, or as something else? I suspect that I need not answer that question in this paper (since Lyon 2012 does not in his paper), but I would allow that the Fourier harmonics are being *instantiated* as Berenstain (2017, §5.3) explains that term.

however, is only the secondary thesis of my paper. My primary thesis is that EMPEC overcomes Saatsi's objections to extra-mathematical program explanation (cf. section 1), an argument to which I now turn.[35]

5. Can Mathematical Properties Program Physical Events?

With EMPEC now articulated, motivated, and defended on a first-pass level, I can argue that EMPEC meets Saatsi's (2016, §5; 2012) objections to Lyon's (2012) approach to extra-mathematical program explanation (cf. section 1). Those objections are threefold: (i) that Lyon's examples like cicadas and Bridges of Königsberg do not fit the PE template, since their explananda patterns are not "events" (Saatsi 2012, 580); (ii) that Lyon's explanantia are nominalizable into non-mathematical terms (Saatsi 2012, 581-582); and (iii) that the explanatory relation of "making probable" or "ensuring" within PE, between mathematical entities and the physical world, is mysterious (cf. section 2).

[35] Many implications and objections to EMPEC must be bypassed in the interest of space. Some might wonder, for example, how the stimulus and manifestation of harmonic-SSR could really be infinitely long. I defer to Morganti (2013, 179) and Barrow (1998, Chap. 6) on the possibility of an infinitely-sized universe, through which infinite-duration light would have room to propagate. On whether a disposition with eternal manifestation would still be a "disposition," see Chakravartty (2013, 45).

Others might press me to explain why harmonic-SSR is not an unreal idealization of reflectance. Pincock (2014), for example, assumes an infinitely-deep ocean to calculate the phase speed of water waves, but he retracts that idealization at the end of his analysis as not ontologically referring; why cannot I do the same with the harmonics of harmonic-SSR? My simple answer is that Pincock faces no *conceptual regress* by withdrawing the infinitudes he posits, whereas I clearly do. This answer applies equally to Ellis's (1992) account of infinite idealization in his epistemology of per-wavelength electromagnetic properties, which I think is mistaken according to my arguments of section 4.

Lastly, to those who object that I fixate on a Fourier reduction base when there are other mathematical methods for representing propagating light, I reply that the onus is on the objector to show which mathematics predicts harmonic dispersion but avoids the VRR; wavelet analysis, for example, fails to block the VRR, since wavelet bases are manifestly heterochromatic like the "envelopes" of Figure 5.2. Nor are Fourier harmonics easily (if at all) nominalizable into anything like Field's (1980) "betweenness" and "congruence" predicates (see Liston 1993, and section 5 below).

Against the first of these objections (i), I appeal to the authority of Jackson's (1998a) PE for color experience, to affirm that seeing color is an event, and that EMPEC thereby fits the program explanation template.

EMPEC also resists objection (ii), I claim, but in a way that most parties to the indispensability debate have overlooked. Firstly, I *agree* with Saatsi that many if not all of Lyon's (2012) examples are nominalizable. With respect to the extra-mathematical program explanation of cicada life cycles, for example, Saatsi denies that primeness explains the 17-year period, since a "fact about time" could instead explain the period: the fact that "[f]or periods in the range 14 to 18 years the intersection minimizing period is 17," conjoined with other premises, entails that the cicadas emerge every 17 years (Saatsi 2016, 1050; cf. 2011, 149). The fact that time divides into units, in other words, and that concatenations of these units can be represented as (e.g.) sticks of various lengths, and that chains of length-17 sticks extend the longest before equaling the lengths of chains comprised of length-14 or length-18 sticks, etc., reveals the evolutionary advantage of a 17-year life cycle without appealing to mathematics, Saatsi (2011, 150 ff.) argues.

As I intimated in footnote 35, however, we have compelling reasons from Michael Liston (1993, §2) to *deny* that the Fourier transform which decomposes pulses into superimposed harmonics is nominalizable into "local" spatial relationships like those expressed by two chains of sticks; the relation of a pulse's amplitude to the amplitudes of its superimposed modes is significantly more complex, and hence impractical to nominalize (if possible at all; 444). Nevertheless, this *difficulty* of nominalizing the Fourier harmonic is *not* my main counter-objection to Saatsi's point (ii). I argue that

EMPEC withstands objection (ii) because EMPEC withstands objection (i). Nominalize the infinite duration of the harmonic into an unreality, or approximate that infinitude into a finitude, and reflectance ceases to be a real property (section 4.1), an absurdity in light of the fact that EMPEC refutes objection (i) as a legitimate PE. The Saatsian cannot argue that dispositional reflectance is unreal *because* Fourier harmonics are unreal, without claiming more than objection (ii) entails; thus, EMPEC survives objection (ii). Whereas a cicada explanation can still be had after Saatsi's nominalization of primeness (if no nominalist explanation could be had, Saatsi would be begging the question about the explanatoriness of primeness), a color objectivist explanation of color experience cannot be had (for all we know) if reflectance is obliterated. The Saatsian can deny that surface colors are real or explain anything, but she cannot do so from the force of objection (ii) alone.

The Saatsian might reply that I can retain the infinite duration of the Fourier harmonics indispensable to reflectance ascription, without that infinitude being *mathematical*; but this suggestion is controversial. Jody Azzouni (1994, 3) maintains, for example, that the infinite is quintessentially mathematical, whereas Hartry Field (1980, 95, 101-102) and René Guénon (2001) defend alternative positions. Prescinding from this elaborate controversy, I conclude that EMPEC is at least not as *trivially* nominalizable as Saatsi takes Lyon's examples to be, and so I resist Saatsi's objection (ii) in this first-pass analysis.

Lastly, EMPEC withstands Saatsi's objection (iii), that the PE relation of "making probable" between mathematics in the explanans, and physical events in the explanandum, is mysterious and scarcely intelligible. If one dips a hollow wire cube into soap solution and removes it, soap film will coalesce within the cube in a repeatable, particular shape; Lyon (2012) program-explains this shape by a mathematical fact (Plateu's laws) about which film geometry minimizes film surface area and hence the potential energy between soap molecules (563-564). Saatsi objects:

> But, of course, no real soap film actually instantiates the *exact* properties investigated by a mathematical theory of minimal surfaces (e.g. geometric measure theory): what we have are idealized mathematical models of real soap films that ignore forces other than those that keep the film together. More importantly, an explanatorily relevant mathematical property can always be instantiated (in this loose sense) in a physical system without being 'realized' in the causally efficacious properties related to Plateau's laws. (For instance, we might craft a model of soap film from some wire and a sheet of copper, also instantiating the minimal surface area for a given geometric configuration.) Thus, such a mathematical property does not *ensure* the instantiation of the relevant causally efficacious properties that ground a lower-level explanation of the shape . . . . (Saatsi 2012, 583)

If I understand Saatsi correctly, Plateau's laws do not "ensure" ("make probable") that *soap* does anything, because Plateau's laws can be just as well approximated in copper sheet and wire; thus there is no "making probable" relation between Plateau's laws and soap. Similarly, Saatsi questions how "primeness" makes probable that cicadas will conduct their lives in a certain way, and how graph theory problematizes the pedestrian's attempt to cross of all of Königsberg's bridges in a continuous sequence, without backtracking any bridge or repeating any crossing.

I reply that EMPEC de-mystifies the "making probable" relation between its explanans mathematics and its explanandum events, in a qualified sense. The qualification is that Fourier harmonics are not themselves explanantia like "primeness" and "graph theory" are; rather, Fourier harmonics "constitute" ascribed SSR_H in the ($D_R \rightarrow SSR_H \rightarrow G$) programming chain, as "pieces of wood arranged table-wise" constitute a table (Dasgupta 2017, 75). While this qualification renders harmonics less explanatorily "punchy" or intuitive than primeness may appear in the cicada PE, that intuition is no threat to my argument, because SSR_H is impoverished (section 2); all parties already agree up-front that SSR_H, while real, does not cause anything, and only "programs" its base property (G) (section 3). Nevertheless, Fourier harmonics "make probable" color experience, because they *make ascribable* the property (SSR_H) that renders plausible the causal efficacy of (G) (section 3). If there are no harmonics, there is no reflectance, and without reflectance, the hypothesis that (G) causes color experience becomes highly improbable.

Here some may object that I am equivocating with an *a posteriori* notion of "making probable," since we know only from extant atomic or electromagnetic theory that surface atoms (of G) must reflect radiation to be observed, whereas the "making probable" that primeness imparts to the cicada explanation appears *a priori*, analytic, or necessarily true. Without taking a position on the tenability of *a posteriori* necessary truths, I dismiss the equivocation charge as unimportant, because of Saatsi's open-endedness about what he would allow the making-probable relation to be. Specifically, he articulates:

> The only option, it seems, is to say that mathematics is involved in the programming relation, not in and of itself, but as an indispensable part of some kind of physical-cum-mathematical property complex. What is such a complex like? How does mathematics get involved in programming via such a complex? I have no idea. (Saatsi 2012, 583)

I propose that "constitution" is one physical-cum-mathematical property complex in which mathematical explanantia make probable explanandum events, and that one illustration of constitution is the manifestation of harmonic-SSR in EMPEC.

Mathematics “gets involved” in constituting the stimulus, instantiation, or manifestation of a programming property, whenever nominalizing the mathematics of that property’s definition launches a conceptual regress (or other fatal problem for the PE). Because program explanations remain explanations even if they fail to epistemically impart any modal information (section 2), Fourier harmonics program-explain color experience despite no one thinking about eternally-durative time when they see a beautiful red car or explain its color (unless it’s *really* beautiful). Granted, this insight about physical-cum-mathematical property complexes follows not from program explanation proper, but from *property realism* more generally, a tenet of PE. Indispensabilists should nevertheless anticipate other such complexes.[36]

6. Conclusion

In this essay, I proposed a counterexample to Saatsi’s (2012; 2016) tripartite objection to extra-mathematical program explanation (PE). My counterexample is Frank Jackson’s (1998a) PE for color experience, which I argue is extra-mathematical for implicitly relying on the reflectance property that suffers conceptual regress unless redefined with Fourier harmonics. I named this counterexample EMPEC, and I argued that it meets Saatsi’s objections because it (i) explains events, (ii) the Fourier harmonics indispensable to its explanantia are non-nominalizable (because of the regress mentioned in the previous sentence), and (iii) EMPEC discloses a minimally esoteric “connection” between mathematics and the world that I call “constitution” (section 5).

[36] E.g., I suspect that Chakravartty’s (2007) dispositional light intensity and Bursten’s (2018) surface plasmon resonance suffer the same “per wavelength” regress as reflectance.

The conclusions to draw from my counterexample are twofold. Firstly, some extra-mathematical program explanations, like EMPEC, withstand Saatsi's criticisms, even if most or all of Lyon's (2012) examples do not. Secondly, the Enhanced Indispensability Argument (EIA) is a superfluous vehicle for mathematical realism in the context of some program explanations, since the property realism of PE reifies some mathematics through "constitution" before the EIA is ever run.

CHAPTER 6

A META-EPISTEMOLOGICAL HURDLE TO MODELING ACTIVE MATERIALS[1]

1. Introduction

A number of philosophical theses motivate the study of active materials like nanoparticles, sub-cellular molecular motors, and microtubules—chemical or biochemical entities whose small-scale properties are manipulated *en masse* for large-scale effects. To demarcate natural kinds, for example, Julia Bursten (2018) argues that the "length, time, or energy scales of [a given] scientific investigation" must be taken into account (3), since (e.g.) elemental "gold" exhibits different colors and reactivities at macro- and nanoscales (§1). Patrick McGivern (2019), on the other hand, hypothesizes that the "autonomous movement, environmental sensing, coordinated action and problem-solving" behaviors of active materials constitute "minimal models of cognition" (2). Minimal models ostensibly explain the "universal pattern[s]" observed between heterogeneous systems, by showing the systems to occupy "the same universality class" as the model (Batterman and Rice 2014, 350). Sometimes a universality class can be defined mathematically, but lacking such a definition at present, McGivern (2019, 9) suggests that non-living systems might explain cognition if they are found to possess universal features of cognitive systems (cf. Needleman and Dogic 2017). Multi-scale

[1] An earlier version of this chapter was submitted to *Synthese* on May 29, 2020.

explanations of other phenomena, like morphogenesis, have also been studied (Green and Batterman 2017).

My contribution to this discussion is to wave a methodological caution flag, for I think that philosophical inquiry into the multiscale modeling of active materials has largely preempted itself. I claim as much because one of the main properties or processes[2] by which researchers measure and test active materials is beset with its *own* multi-scaling problem: conceptual regress at all time scales. The property in question is surface plasmon resonance (SPR), a form of electromagnetic propagation utilized by researchers of nanoparticles (Choudhary et al. 2017, 38; Zeng et al. 2011, 492 ff.), motor proteins and microtubules (Needleman and Dogic 2017; Hasegawa et al. 2019; Young et al. 2003), and colloidal suspensions (Bhattarai et al. 2017, §5.1; Lupusoru et al. 2016), and appealed to in Bursten's (2018) philosophical analysis of nanoparticles (2, 8, 14-15). The *technique of measuring* SPR has matured for several decades (cf. Pockrand 1978), so a specific SPR technique is not the focus of my critique, nor would a change in technique allay my concerns. My criticism pertains rather to the long-used and commonplace optical properties that partially constitute SPR, properties like reflectance, absorbance, and refractivity.

In this paper, I argue that the received view of reflectance (to analyze the simplest example) suffers conceptual regress and incoherence at all timescales,[3] and hence so does SPR. This incoherence about SPR, in turn, renders multiscale modeling epistemically

[2] I remain neutral on property realism/antirealism, and sometimes call a "property" what might be better understood as a process. A number of multiscale modelers discourage ontological commitment to properties (McGivern 2008, 65; Wilson 2018, 237).
[3] I first detailed this argument in Danne (2020).

dubious, since the macro- and nanoscale properties ascribed within a model are themselves discovered, studied, or measured by optical properties (reflectance, SPR) whose definitions turn out to be meaningless and even self-contradictory. This inter-scalar regress about the properties (reflectance, SPR) *used to study* inter-scalar properties of active materials I characterize as a meta-epistemological "hurdle" to active materials modeling. The hurdle deserves explicit attention from multiscale modelers, moreover, because the hurdle's nearest-to-hand solution resembles a clear non-solution with which scale modelers are familiar: homogenization, or the blowing-up of a modeling parameter to infinity to facilitate information transfer between scales. In sum, the simplest way to save SPR from conceptual regress is to transform it into what looks like a homogenization; but since SPR cannot be a homogenization (for reasons to be explained), multiscale modelers need to block the regress in a different way, or justify allowing it. I argue that a particular account of scientific fictional modeling (Purves 2013) is an unhelpful alternative to homogenization, and that fictional modeling generally seems misguided as a solution to the meta-epistemic "hurdle."

In section 2, I provide background on SPR, and I analyze its regressive incoherence in section 3. I examine solutions to the SPR regress in section 4, and conclude in section 5.

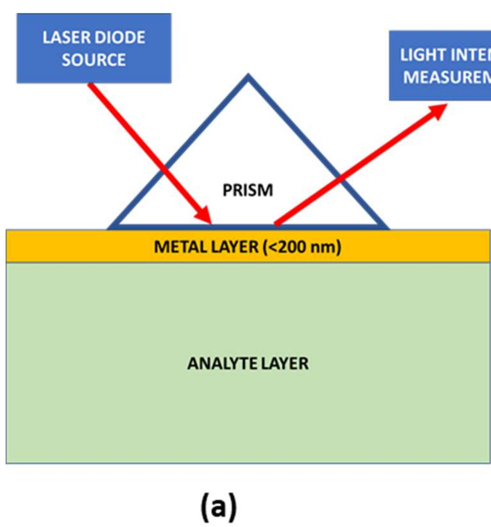
2. A Primer on Surface Plasmon Resonance

According to Yuriy Ushenin, Volodymyr Maslov, and Glib Dorozinsky (2017):

> Surface plasmon resonance is a phenomenon that involves absorption of p-polarized light by the surface electrons of a metal film (e.g., silver or gold) under specific resonance conditions defined by the dispersion relations of surface plasmons [. . .]. This resonance condition is extremely sensitive to the refractive index and thickness of dielectric medium (for instance,

liquid, gas, or solid) in contact with the metal surface. (Ushenin et al. 2017, 3-4)

Said another way, SPR is the tendency of a metal-dielectric interface to propagate longitudinal electromagnetic waves into the dielectric (Shalabney and Abdulhalim 2010, 24), a propagation commonly initiated by a laser pulse through a refracting prism (Ushenin et al. 2017, 4, 6). If the angle and wavelength of the laser are just right, and the metal film "sufficiently thin (<200 nm)," then SPR "becomes sensitive to the properties of the medium" on the far side of the metal (Ushenin et al. 2017, 6). Active materials researchers locate their experimental analyte on this far side of the metal, to infer from SPR the analyte's small-scale properties, such as the concentration of an antibody in blood or milk (Ushenin et al. 2017, §4.1).



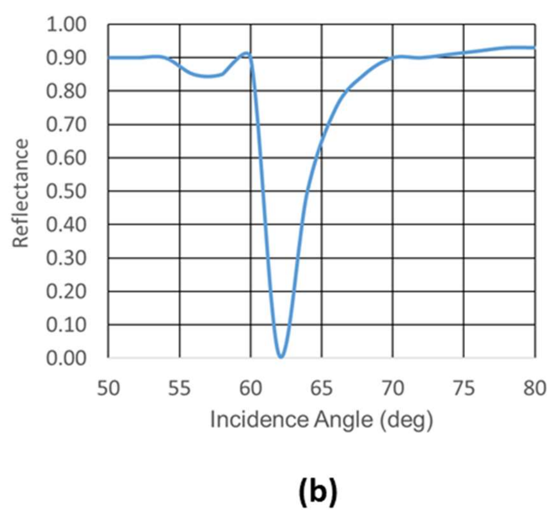


Figure 6.1: (a) Simplified SPR setup (not to scale); (b) Hypothetical SPR profile for λ = 650 nm light

An optical indicator by which researchers infer SPR to be occurring is a sharp dip in the intensity of laser light reflected from the metal film (and back through the prism, see Figure 6.1). This dip in intensity has quantified significance only in relation to the intensity of the laser source, and so researchers sometimes plot "reflectance," or the efficiency between 0 and 1 of a surface to reflect light, rather than prism output intensity alone (Figure 6.1 (b)). The metal film stops reflecting, that is, when it begins plasmon-resonating, and the angle of resonance may change with analyte. Per the antibody example of the previous paragraph, Ushenin et al. (2017) found the angle of resonance to increase with antibody concentration (23).[4]

More advanced versions of SPR, known as localized surface plasmon resonance (LSPR), dispense with the prism and metal layer of Figure (a) (Bhattarai et al. 2017, 89), and measure the extinction spectrum of noble metal nanoparticles experimentally situated within microbiological structures. The extinction spectrum is a function of reflectance, and like reflectance, is measured with a spectrophotometer (Hasegawa et al. 2019, 2), so for simplicity, the conceptual regress that I launch against SPR and LSPR in section 3 is just a regress about reflectance generally construed. In the meantime, the point to grasp is that LSPR researchers, like SPR researchers, optically sweep the analyte to discriminate its small-scale properties. Keisuke Hasegawa, Otabek Nazarov, and Evan Porter (2019), for example, report successful detection of microtubule nucleation *in vitro*, using LSPR and gold nanoparticles. Why this heavy reliance on reflectance throughout multiscale modeling should give anyone pause, I explain next.

[4] The dip in reflectance profile can be alternatively measured by holding laser angle constant and varying wavelength, or—as in Figure 6.1 (b)—by holding wavelength constant and varying incident angle.

3. The Multiscale Conceptual Incoherence of SPR

The problem with the reflectance inherent to SPR and its measurement is that reflectance is a "per wavelength" property of materials, namely the per-wavelength efficiency of a surface to reflect finite-duration pulses[5] of light—to use the most philosophically influential, scientifically-informed definition (Hilbert 1987, 1037-1041; also Byrne and Hilbert 2003). As I argue in Danne (2020), however, this definition overlooks a well-documented[6] empirical law of nature that problematizes reflectance ascription: that law I call "harmonic dispersion," the inverse relation of a pulse's duration to its bandwidth.

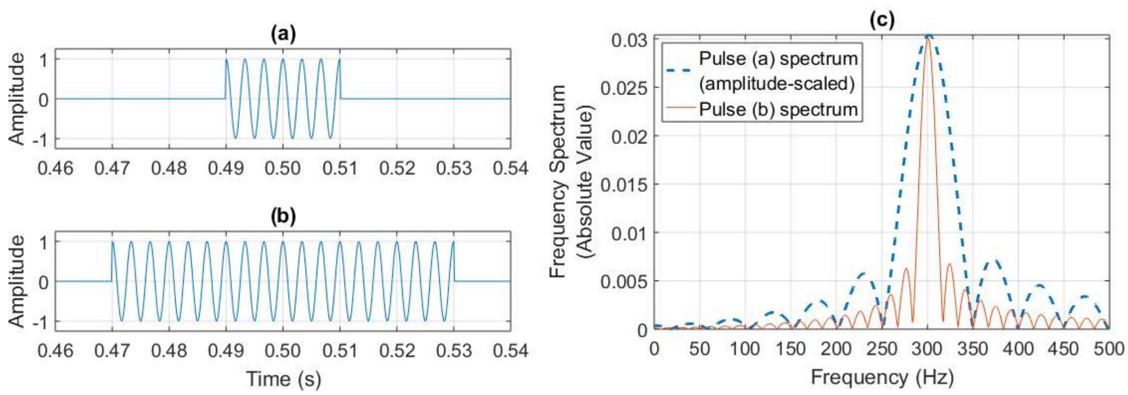


Figure 6.2: Harmonic Dispersion [3x duration]

Figure 6.2 depicts harmonic dispersion, revealing that two pulses with center frequency[7] of 300 Hz and a threefold difference in duration dissipate considerably different percentages of their pulse energy at 300 Hz; whereas the longer pulse focuses most of its

[5] By "pulses," I mean transverse waves of (co)sinusoidal, Gaussian, or other common form. For examples, look ahead to Figure 6.2 (a) and (b).

[6] By Haykin (1989, 37), Stingl et al. (1995), and Deng et al. (2005), to name a few of very many.

[7] Reflectance is a "per wavelength" property, but wavelength and frequency are interchangeable in physical optics: frequency is speed of light divided by wavelength.

energy at 300 Hz (plot (c) solid trace), the shorter pulse dissipates significant side-peaks of energy at 225 Hz and 375 Hz. One could say that the longer pulse propagates "per wavelength" *better* than the shorter pulse does, because it comes closer to propagating at the "wavelength" which entered the analysis: 300 Hz.[8] An equivalent insight is that truncating a monochromatic pulse introduces auxiliary frequency content, in inverse proportion to the pulse's duration at truncation.[9] But again, this auxiliary content should not be interpreted as "instrument noise," medium distortion, or any quantum duality or uncertainty misapplied to the macro-domain (Hirlimann 2005, 31); harmonic dispersion obtains in even the cleanest real signals with macro-sized millisecond durations.

By this observation alone, *viz*., that harmonic dispersion afflicts all usable pulse durations and frequencies,[10] the regress of the pulse-reflectance concept may be generated. One simply need follow the assumption of the received view, that reflectance is the per-wavelength ratio of reflected to incident average powers of light at a surface (Hilbert 1987, 1037-1041). Laser researchers depict harmonic dispersion akin to Figure 6.2 for optical pulses of sub-picosecond (ps; 10^{-12} seconds) durations (Deng et al. 2005; Stingl et al. 1995), so we may imagine that a 5 watt (W) pulse from a 650 nm monochromatic source does not reflect all 5 W at the surface of a perfect mirror (reflectance ratio = 1), at 650 nm. Glancing at Figure 6.2, we might instead estimate that 80% of the 5 W—namely 4 W—reflects at 650 nm, while the other 20% reflects at

[8] Light propagates at hundreds of trillions of Hertz, but the lower-frequency toy model of Figure 6.2 is easier to program and display. The wavelength of a 300 Hz electromagnetic signal is 1,000 km.

[9] Mark Wilson (2018, Chapter 5, §iii) discusses and illustrates this point.

[10] Stingl et. al (1995) depict harmonic dispersion for the femtosecond (10^{-15} second) pulses used in laser applications. Haykin (1989, 36-37) reports harmonic dispersion for much lower-frequency and longer-pulsed radar applications.

auxiliary wavelengths surrounding 650 nm (the accuracy of this eyeballed estimate will not matter, because a regress is a regress).

But now we must ask: will 4 W *really* reflect *entirely* at 650 nm? The question is not about real-world distortion, noise, or heat loss, but about harmonic dispersion. If 650 nm, sub-picosecond pulses disperse 20% of their average power (see previous paragraph), then will not the 4 W "component" of the original pulse—the component *really* propagating at 650 nm—disperse 20% of its average power, too? If so, then what really propagates at 650 nm is 3.2 W (80% of 4 W), but by the same argument, what really propagates at 650 nm is 2.56 W (80% of 3.2 W), and so on, *ad infinitum*. The reflected and incident pulse average powers at the mirror both regress asymptotically to zero, and so what we called the perfect mirror, a surface with reflectance value 1 at 650 nm, actually possesses a value asymptotically approaching 0/0 at 650 nm, an undefined and meaningless value! Hence my epistemic "hurdle" for the multiscale modeling of active materials: if the reflectance and SPR utilized for such modeling has the value of ~0/0 at all wavelengths, then how can SPR teach us anything about active materials?

Important to note is that I have not shown this reflectance regress to be a *physical* regress; the scientist measuring a "5 W" pulse probably records 4 W at 650 nm and thinks nothing further of it. But whatever the scientist measures, it is not a *pulse*, by the argument just given. What, then, can be reflecting at 650 nm? What, ontologically, is the scientist measuring? Clearing the meta-epistemic "hurdle" to active materials modeling requires answering that question, lest philosophers perpetuate the unhelpful implication that a property with value 0/0 informs us that gold has different reactivities or colors across various scales of size or mass.

Before considering what besides pulses could be reflecting at 650 nm, I should address the objection that I am being unfair to the received view of reflectance (and by extension SPR), since other credentialed and usable scientific properties lack complete definitions. Peter Smith (1998, 39-41), for example, points out that an object's temperature, defined as molecular mean kinetic energy (MKE), is meaningful and expressible to only a few decimal places, since the imaginary sphere through which molecular velocity is estimated shrinks with every decimal place, until the sphere is smaller than the molecules in question and molecular velocity becomes undefined. In reply, I point out that Smith's problem of adding decimal places to the right, and thereby increasing the accuracy of the temperature estimate until a limit is reached, is a good problem to have. The reflectance regress does something different: it obliterates the reflectance ratio entirely, by sending it asymptotically to 0/0. Even if the value 0/0 is never actually reached, the "per-wavelength pulse" concept has been rendered practically useless and contradictory, since "pulses" do not propagate or reflect "per wavelength."

4. A Solution for Blocking the Reflectance Regress (that Multiscale Modelers Already Use, but Differently)

4.1 Defining and Ascribing Harmonic-Reflectance

One straightforward solution to the reflectance regress, in my view, is to redefine "pulses" from finite-duration sinusoidal propagations of light, to the superposition of weighted and phase-shifted Fourier harmonics that sum without remainder into pulses. Harmonics are infinite-duration monochromatic sinusoids, and thus tend to be regarded as more mathematically abstract than physically real (Wilson 2018, 233, 238; McGivern 2008, 68, n. 19; Liston 1994, 18). That such a superposition cleanly models pulses, however, I have already shown. Every point of the solid or dotted trace in Figure 6.2 (c) indicates the amplitude and frequency of a Fourier harmonic of infinite duration, which superimposes with its neighboring harmonics into the respective pulse shown in Figure 6.2 (a) or (b).[11] Most importantly, harmonics are by definition *immune* to harmonic dispersion, since the ratio of their duration to their bandwidth is always unity. When a scientist measures a "4 W pulse at 650 nm" (see section 3), in my account, she is measuring that finite-duration portion of a 650 nm harmonic that is always propagating, a harmonic whose neighboring harmonics destructively interfere with it only outside the times of the respective pulse duration.[12] Thus, no matter the duration of a pulse impinging on a surface, per-wavelength harmonic-reflectance remains a stable, conceptually coherent property, unlike per-wavelength pulse-reflectance, which as we saw in section 3 is a contradiction in terms.

4.2 Harmonics and Homogenizations

Should multiscale modelers accept the harmonic-reflectance solution to the reflectance regress? One may find it strange to claim that the reflection of infinitely-durative harmonic light (in SPR setups) is what reveals to us some of the different properties and reactivities of "gold" at different size scales. I counter that this claim is no *stranger* than analyzing gold according to the received view of reflectance, in which the metal foils employed in SPR reflect with efficiency 0/0 at every wavelength (section 3).

[11] The per-wavelength phase plot for Figure 6.2 is omitted for clarity.

[12] By analogy, the 300 Hz pulse (a) in Figure 6.2 should be construed as a portion of the 300 Hz harmonic of graph (c); for the time period between 0.49 and 0.51 seconds, that harmonic is not cancelled-out, but outside that time period, the 300 Hz harmonic incurs destructive interference from its harmonic neighbors weighted and phase-shifted in just the right way (partially indicated by graph (c)). Such is how Fourier analysis works and applies to the physical world.

Perhaps more profitable than a clash of intuitions, however, is the observation that multiscale modelers already employ infinitely-sized parameters in their models, not to pick out existents, but to abstract large-scale parameters or "dominant behavior[s]" of interest from small-scale data (Wilson 2018, 218). Could my solution to the reflectance regress be just another of these parameter blowups? I doubt it, but let us consider why.

Mark Wilson (2018) calls the infinite blowups of model parameter values "exaggerated infinity cures" (217), and he gives the example of assuming an infinite number of gamblers to obtain a Gaussian distribution about probable wagering behavior at casinos (217-218). Somewhat closer to the reflectance problem, moreover, Wilson models the elasticity of a macroscopic metal bar (with moduli E and μ), pointing out the "greediness" of differential equations needed at any given scale length of the bar. These equations are greedy because they must range into the "infinitesimal" to work properly, even though the elasticity scale of interest is much larger than the infinitesimal (214). The intermediate, unwanted, result of this conceptual-mathematical tension is an "intractability" of scale-dependent elasticity parameters (214), since the equations modeling behavior at one scale incidentally extend into a lower scale, where they confute the description of conceptually different and non-reducing properties (203-204).

A solution to this greediness, Wilson (2018) explains, is another "infinity cure," what modelers call a "homogenization," and which itself takes the form of differential equations extending into the infinitesimal. The purpose of this homogenization is not to calculate E and μ from the bottom up (since E and μ do not obtain in spatial regions shorter than 10 um;[13] see p. 202), but to iteratively predict when steel grain dislocations

[13] 1 um = 1 micrometer (10^{-6} meters)

will have migrated due to repeated force on the bar, such that the macroscopic E and μ values of the bar should be updated (chap. 5, §iv). Hence Wilson metaphorizes homogenizations as a "Morse code" (226) between modeling levels that partially explain a macro-property (bar elasticity), despite the absence of any univocal property referent between the levels (since lower-scale dislocations do not possess E and μ). The homogenization mathematics, in Wilson's view, neither pick out an ontology of real entities (233, 237), nor "'captur[e] the entire truth' about nature's relevant activities" (234); the mathematics rather "imitate the cross-scalar dependencies" (232) within the system of interest, and Wilson counterposes this "imitation" thesis to the Quinean practice of existentially quantifying over the posits of our best first-order regimented theories (chap. 5).

In reply, my reason for using an infinite blowup seems entirely different from Wilson's (2018). I am trying to prevent a vicious definitional regress of one property, reflectance, which by all appearances in scientific and philosophical literature, is time-scale *independent*.[14] I am not attempting to update long-duration SPR values with short-duration laser reflectance data (whatever that suggestion even means), and I am not trying to overcome the intractability of reflectance at a particular scale. I instead attempt to render reflectance a conceptually coherent and ascribable property at *any scale* by making harmonic-reflectance the trans-scalar univocal referent ostensibly employed throughout the model (the univocal referent that E and μ could not be in the elastic bar

[14] Of course, some active material surfaces might reflect or be catalyzed by only (e.g.) femtosecond light, but this contingency does not entail that reflectance itself—an efficiency ratio like 0.5—is temporally scaled. The quantity 0.5 is 0.5, lacking any temporal variable whereby a timescale shift would change the meaning of "0.5." Indeed, how could 0.5 efficiency ever mean something different than 0.5 efficiency?

example). For we have little reason to suppose that reflectance itself suffers an inter-scalar dependency (see footnote 13). Thus, the harmonics as I employ them do not resemble the homogenizations as which Wilson classifies his differential equations.[15]

4.3 Dispensable Mathematics and Scientific Fictions

One might attempt to dodge the infinite blowup of harmonic duration altogether, by objecting that my gravitation to a Fourier representation of light propagation is unnecessary. Wavelet analysis, for example, does not decompose pulses into harmonics, but correlates pulses to fundamental sub-pulses, or wavelet bases. While this correlation is generally more efficient than a harmonic superposition, the wavelet representation does not block the reflectance regress philosophically. Indeed, because wavelet bases are themselves finite-duration pulses, representing or explaining harmonic dispersion (and "per-wavelength" reflectance) in terms of them results in conceptual regress (section 3).

Alternatively, one might avoid harmonic realism by interpreting harmonics as useful fictions with predictive power (about harmonic dispersion and SPR) that do not reference real entities. One advocate of these scientific fictions is Gordon Purves (2013), who describes them as

> those elements that, when introduced into a theoretical model, fail to fit into a monotonic series of improvements, and yet are still truth conducive. In other words, fictions are added to or incorporated into models to aid in producing accurate predictions, but are themselves more unrealistic than some tractable alternative which produces worse predictions. (Purves 2013, 245)

[15] I similarly doubt that the reflectance regress can be blocked by applying the "fast" and "slow" timescales of perturbation theory discussed by McGivern (2008, §3). For that solution pertains to the application of complex differential equations to physical setups, which I am not doing.

Applying this advice to the reflectance regress requires first understanding what needs to be "improved." If predictions about harmonic dispersion need to be improved (a real scientific need unrelated to the reflectance regress), then harmonics seem to fit Purves's account. Longer pulses monotonically superimpose better (by Fourier composition) into the finite-duration pulses that disperse, and infinite-duration pulses (harmonics), which are allegedly unreal, produce the best predictions of all.

But I am not trying to monotonically improve my predictions about harmonic dispersion; I am trying to represent reflectance, at any wavelength, for any pulse duration. With respect to this task, monotonic "improvements" seem to be neither here nor there. For a first improvement from the 0/0 reflectance regress in 80% iterations (section 3) would be a regress of smaller iterations (e.g., 99%), conducted with longer, less-dispersive pulses. But this "improvement" is illusory, because the 99% regress is just as vicious as the 80% regress, resulting in reflectance values of 0/0 at every wavelength. There appears to be no "tractable alternative" to the harmonic, for solving the reflectance regress, and so fictionalizing the harmonic as a truth-conducive non-entity seems premature.

5. Conclusion

In this note I have argued that multiscale modelers of active materials face a meta-epistemological difficulty in explaining how they know and learn about the properties of entities occupying small-scaled regions of time and space; for one of their main conceptual and physical laboratory tools for that investigation is optical reflectance and surface plasmon resonance (SPR), two concepts whose received definitions I have argued to be regressive and meaningless. One of the most expeditious routes to blocking the reflectance regress is introducing an infinite blowup of optical pulse duration to infinity; while this infinitude may appear ontologically implausible, I argued that it is not obviously nominalized as a homogenization, or as the fictional element of a model, according to one account of fictional models in science (Purves 2013).

Granted, there are many accounts of fictional models that I have not analyzed, but the more pressing question is whether any such models are or should be employed to represent a property as fundamental to science as reflectance is supposed to be. Purves (2013) introduces fictional cracks into media to predict real fissures, but the cracking of media is ostensibly several orders more complex than the reflection of electromagnetic radiation at a given spatiotemporal scale is supposed to be, since reflectance is just a ratio of basic quantities (average power, intensity, etc.). I conclude that multiscale modelers of active materials should explicitly address the reflectance regress, resolving it or excusing it, lest they fail to clear the meta-epistemic hurdle of explaining inter-scalar properties *by means of* a property (reflectance) with its own inter-scalar incongruities.

CHAPTER 7

IS FOURIER ANALYSIS CONSERVATIVE OVER PHYSICAL THEORY?

I.*

Hartry Field (1980, 13) famously argues that the mathematics suitable for application to science should be conservative, in the sense that conjoining mathematics to nominalist science should yield no conclusions not derivable from nominalist premises alone. This conservativeness requirement advances Field's nominalism about mathematics, by eliminating the supposed need for applied mathematics to be true and referential. A consistent and conservative mathematics could be strictly false, while facilitating inferences between claims of nominalist science, just as the conjunction of $((A \rightarrow B) \& A)$ yields B, even if $(A \rightarrow B)$ is never in fact true in its world of application. In this paper, I argue that the branch of mathematics known as Fourier analysis, which enjoys ubiquitous application throughout science and engineering, is non-conservative over physical theory. My objective is not to argue that Fourier analysis should be construed as true or false,[1] but only to present a succinct counterexample to Field's conservativeness claim.[2] In turn, I offer this counterexample as one starting point for discussions away from nominalism

*In the short paper on which this chapter is based, I did not use section titles, and would find it artificial to invent some now. I submitted a slightly revised version of this chapter to *Thought: A Journal of Philosophy* on March 12, 2021.

[1] I allow that mathematics could be true, false, or neither, for the purposes of this paper.

[2] Others, such as Melia (2006), have criticized Field's conservativeness requirement without proposing positive counterexamples. Liston (1993, 444) suspects that Fourier analysis is non-conservative over physical theory, but again works out no concrete example.

and toward a mathematical realist position, although I offer no *defense* of realism in this paper.

For terminological convenience, I adopt Kenneth Boyce's paraphrase of Fieldian conservativeness:

> **Conservation:** For any background mathematical theory, MT*, that is suitable for use in science, the conjunction of MT* with any body of non-mathematical statements, N*, has as logical consequences all and only the same non-mathematical statements as does N*. (Boyce 2020, 13, formatting mine)

From this definition, we see that conservativeness is a *logical* property, although what is meant by logical, as in *a priori* or necessary, is left somewhat open.[3] I exploit this ambiguity in section V, by arguing that the non-conservativeness of some (mathematical) statements is a contingent function of agential knowledge or psychology. In the meantime, I should clarify what I mean by proposing "Fourier analysis" to be non-conservative. I focus on a precise operation within Fourier analysis known as the Fourier transform, which is the superposition of weighted and phase-shifted "harmonics"—infinite-duration monochromatic sinusoids—into finite-duration pulses[4] that obey Dirichlet conditions.[5] The transform operates between the frequency and time domains of analysis: harmonics allow temporally extended pulses to be described entirely in terms of weighted and phase-shifted frequencies.

[3] Even Field's (1980) chapter 9, "Logic and Ontology," does not elaborate what the nature of logic is.

[4] See Figure 7.2 (a) and (b), below, for examples of "pulses." Laboratories generate and measure pulses with profiles more complex than the simple sinusoids of Figure 7.2, but this difference has no bearing on the arguments in this paper.

[5] Dirichlet conditions fall outside the scope of this paper (see Haykin and Van Veen 1999, 172).

To show that Fourier analysis is non-conservative, I will argue that adding Fourier-mathematical sentences to nominalist sentences N* about physical reality entails nominalist sentences[6] about physical reality not derivable from N* alone. Particularly, I argue that nominalists studying light propagation require Fourier mathematics to assert nominalist claims about surface reflectance. Such nominalist claims include, "This surface is reflective," and, "Surface A is more reflective than surface B." For brevity, I call these claims "Reflectance Claims."

II.

To show that Reflectance Claims are nonsensical in nominalist science, I introduce a thought-experimental world named "Shineland," whose inhabitants speak a mathematics-free English, and who have developed physical theory N*. Shinelanders are simple folk, in that they occupy themselves mostly with studying the "average power" of light (and microwaves, radar, radio, x-rays) that impinges on surfaces, and with the average power of light (et. al) that propagates away from those surfaces after they are impinged. In mathematized English, the average powers of finite-duration electromagnetic impingements and radiations (*viz*. transverse "pulses") are:

$$P_{avg} = \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt \qquad [1]^7$$

where T is the fundamental period of the propagating signal, x is the amplitude of that signal, and t is time. The units of average power are watts, but Shinelanders do not refer to numerically quantified watts (e.g., a 5 W pulse). Shinelanders nominalize pulse power values into wattages that are **between** others, and **congruent** with still others, in the

[6] I treat sentences, statements, assertions, and claims as the same thing.
[7] Equation 1 is from Haykin and Van Veen (1999, 21).

Fieldian (1980) sense.[8] Indeed, Shinelanders also nominalize the average power integral (Equation 1), replacing any reference to divisions by "2" and powers of "2" within it, as well as any mathematical profiles of the signal like $x(t) = \cos(\omega t)$;[9] the details I must bypass in this paper.

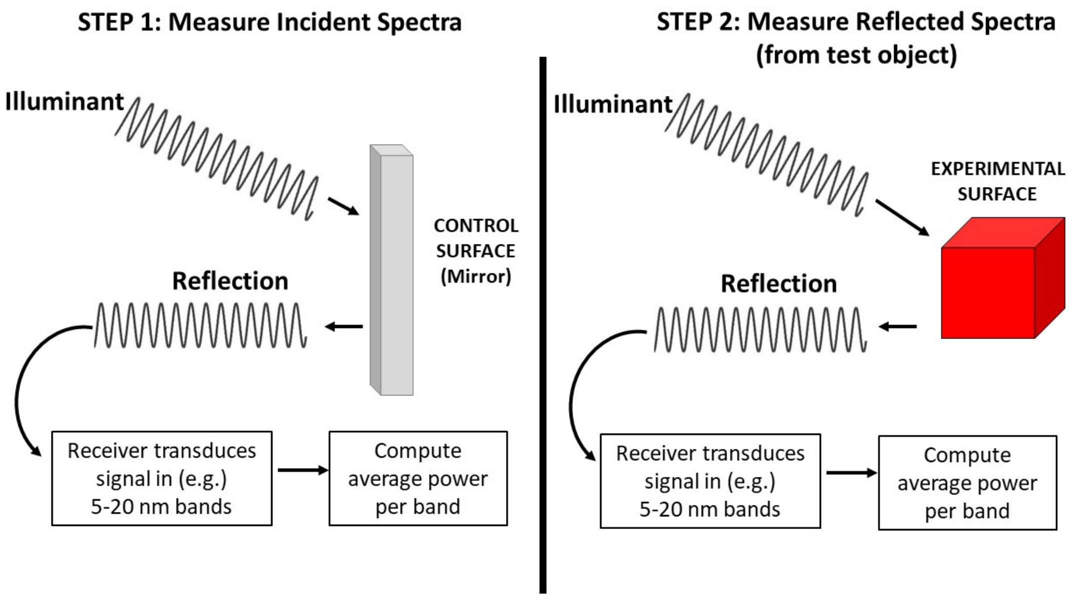


Figure 7.1: Pulse Average Power Measurements

Figure 7.1 depicts (in mathematized English) how Shinelanders (and Earthlings) measure pulse average power,[10] *en route* to measuring reflectance. The reflectance of the red cube in Figure 7.1, for example, is the *ratio* of the average power measured in "Step

[8] Very roughly, Field's (1980) nominalism expunges natural numbers from science by pointing out that one need not say that a pot of boiling water is "100 °C," if one can instead specify its temperature as lying **between** one object's temperature and another object's temperature, or as possessing a temperature difference with another object that is **congruent** with the (non-numeric) difference in temperature between two other objects.

[9] ω represents angular frequency, which is 2π times the center or carrier frequency (in Hertz) of a simple (co)sinusoid.

[10] Figure 7.1 depicts an approximate workflow for spectrophotometry, which measures fairly long-duration (microsecond to few-second) light pulses. Other kinds of pulses need different equipment for their detection, which falls beside my purposes to describe.

2" to the average power measured in "Step 1." This ratio is computed "per wavelength" across the human-visible band (400-800 nm), and the red cube's reflective efficiency likely peaks around 650 nm. Given Equation 1, therefore, and the Shinelanders' nominalization techniques, Reflectance Claims about the cube and the mirror (in Figure 1) seem unobjectionable. Why do Shinelanders nevertheless reject Reflectance Claims as nonsensical, and refuse to utter them?

The answer is that, when all you do is study the "average powers" of finite-duration pulses, you begin to realize that there is no sound way to define the "reflectance" property[11] that interests you. Through their daily scrutiny of electromagnetic pluses, the Shinelanders invariably notice what I call "harmonic dispersion": the empirically confirmed[12] inverse relation of *any* pulse's duration to its bandwidth. It is the reality of harmonic dispersion, I will argue in section III, that renders Reflectance Claims nonsensical, when reflectance is defined in terms of pulse "average powers." (This definition of reflectance is the philosophical view championed by David R. Hilbert (1987; Chap. 3) and Byrne and Hilbert (2003),[13] and in this paper I present a metaphysical criticism of that influential received view.[14])

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[11] My argument remains neutral between property realism and property nominalism. Shinelanders might be Humeans who believe only in a "constant conjunction" between impingements and radiations, and who disbelieve in real properties. I proceed in terms of property realism for convenience of prose.

[12] That is, empirically confirmed by contemporary scientists on Earth, across many scientific domains.

[13] Hilbert's (1987) account of reflectance continues to be defended by Byrne and Hilbert (2004; 2007). They take the controversial step of ontologically reducing human-visible color *to* reflectance, but I need not do so.

[14] My fuller criticism appears in Danne (2020).

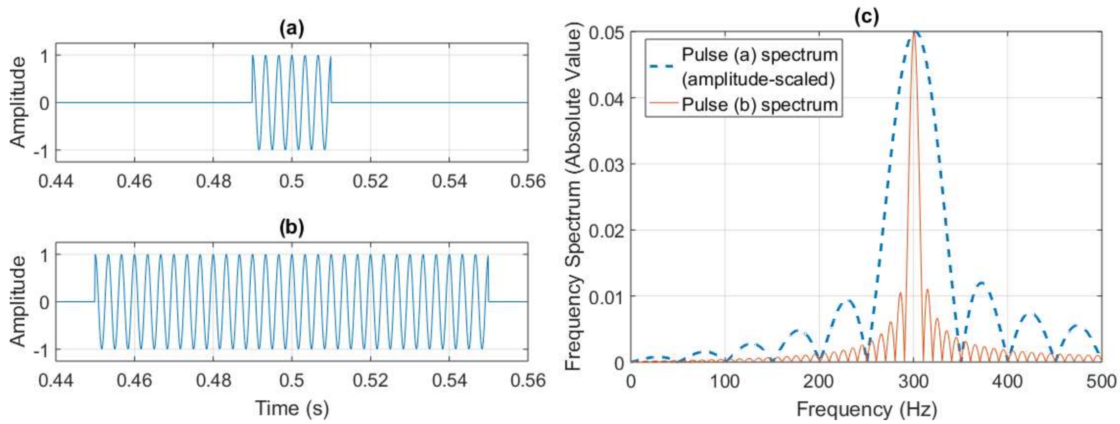

Figure 7.2: Harmonic Dispersion [5x duration]

Figure 7.2 depicts harmonic dispersion in mathematical terms, for pulses (a) and (b) with arbitrarily low carrier frequency of 300 Hz (for ease of modeling). The point to notice is that although input signals (a) and (b) enter the analysis as monochromatic, differing only in their durations, the spectral content of the two signals differs considerably (graph c).[15] The longer the duration of a monochromatic pulse, the narrower its bandwidth (as the solid-trace profile is narrower than the dotted profile, in graph c). Importantly, this inverse relationship should not be misconstrued as any anomaly like "measurement noise," a mathematical artifact, or a quantum-physical duality or uncertainty misapplied to the macroscopic domain. Harmonic dispersion is instead a mainstay of classical (Maxwellian) physics (Hirlimann 2005, 31) confirmed in the laboratory (Stingl et al. 1995; Deng et al. 2005).

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[15] The "spectral content" in graph (c) is the Fourier transform of finite-duration pulses (a) and (b):
$G(f) = \frac{AT}{2}\{\mathrm{sinc}[T(f - f_c)] + \mathrm{sinc}[T(f + f_c)]\}$, for pulses of amplitude A, period T, and center or carrier frequency f_c (Haykin 1989, 37. *Ibid*., 18, defines $\mathrm{sinc}(x) = \sin(\pi x)/\pi x$). For clarity, I omit from Figure 7.2 the negative-frequency harmonics generated by "sinc[$T(f+f_c)$]."

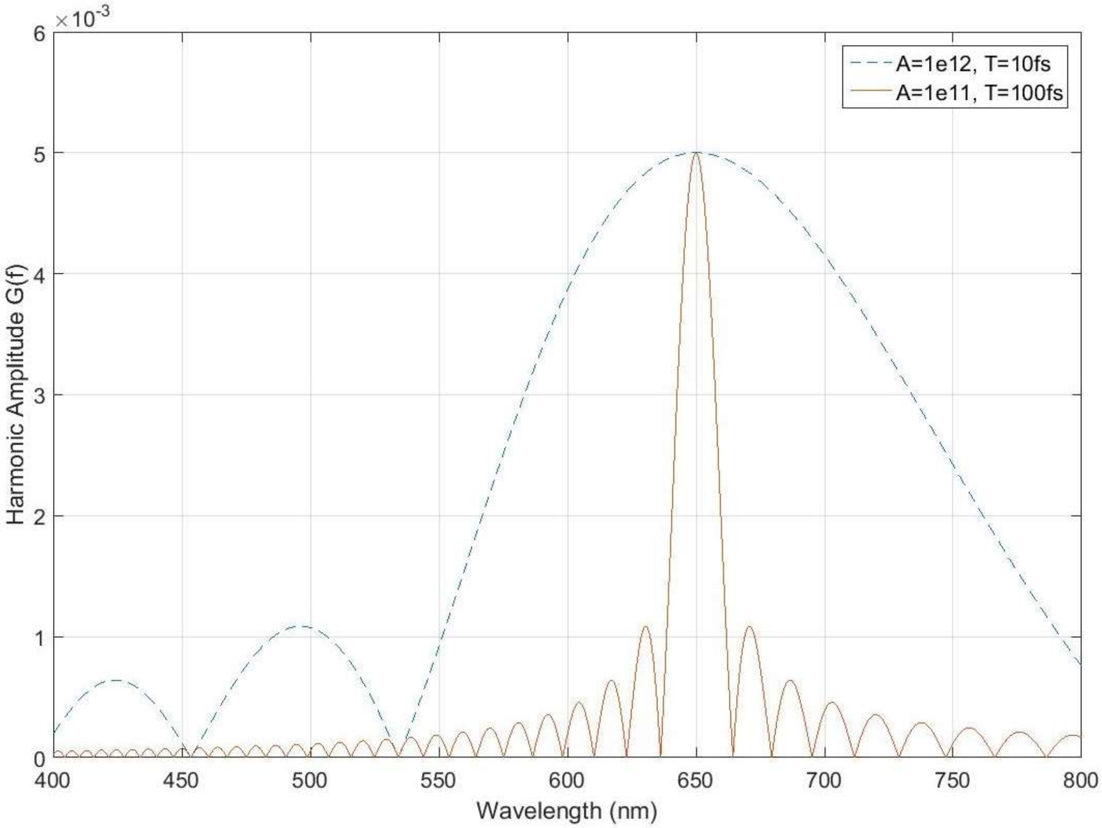

Figure 7.3: Sub-picosecond Harmonic Dispersion[16]

Figure 7.3 depicts harmonic dispersion values that I will utilize in the next section, to undermine the received view of Hilbertian reflectance. The solid trace depicts the spectrum of a 100 femtosecond (fs; 10^{-15} seconds) pulse, and the dotted trace depicts the spectrum of a 10 fs pulse, both pulses centered at 650 nm.[17] Important to notice is that the dispersion of the shorter pulse (dotted trace) is considerable, exceeding 100 nm bandwidth, as laser laboratories corroborate (Stingl et al. 1995; Deng et al. 2005). This

[16] This figure is a reproduction of the Fig. 2 from Nicholas Danne, "How to make reflectance a surface property," *Studies in History and Philosophy of Modern Physics* 70 (2020): 19-27, https://doi.org/10.1016/j.shpsb.2020.01.002 (cc) BY-NC-ND, used with permission in this dissertation.

[17] Actual trillion-volt signals are not assumed for my analysis; I select the pulse amplitude values ("A") in Figure 7.3 for aesthetic depiction.

high-bandwidth dispersion, I will argue, renders "per-wavelength" pulse-reflectance conceptually incoherent, and Reflectance Claims meaningless.[18]

III.

In this section, I detail my argument that reflectance cannot be the per-wavelength efficiency of a surface to reflect pulses (the received view). The argument proceeds readily (and closest to Hilbert 1987) in terms of pulse "average powers," but could proceed alternatively in terms of instantaneous power, or pulse energy. Nevertheless, in their mathematically nominalist fashion, Shinelanders realize that a 5W pulse generated from a 650 nm monochromatic source does not reflect all of its power at 650 nm (assuming the reflection surface to be a perfect mirror). According to Figure 7.3 and equivalent empirical findings, a sufficiently short-duration pulse dissipates considerable energy at 492 nm, as well as at the 650 nm center wavelength. For simplicity, let us say that 80% (4W) reflects at 650 nm, and 20% (1W) reflects at 492 nm. Note that the accuracy of this estimate matters little, because I am going to argue that pulse-reflectance (the received view) is incoherent *for being* conceptually regressive. The reflectance value at any wavelength, in other words, regresses inevitably to the ratio 0/0, a value that is undefined and meaningless, and therefore not the value of a real surface property.

Here is how the regress goes. We have already seen that a 5W, 650 nm pulse actually reflects only 4W at 650 nm. Now the crucial question: does this 4W component reflect all of its power at 650 nm (assuming a perfect mirror)? According to the received view, the answer must be yes, because the 4W component has the same duration as the

[18] Actually, *any* non-unity bandwidth dispersion renders Reflectance Claims incoherent, but the 100 nm bandwidth example may be the easiest to follow.

5W initial pulse, and is therefore itself a 650 nm pulse. But I say no, because harmonic dispersion *just means* that finite-duration pulses disperse their frequency content (Figure 7.2). Thus the 4W component only dissipates 3.2W (80% of 4W) at 650 nm, and that 3.2W component at 650 nm likewise disperses its average power to neighboring frequencies, *ad infinitum*.

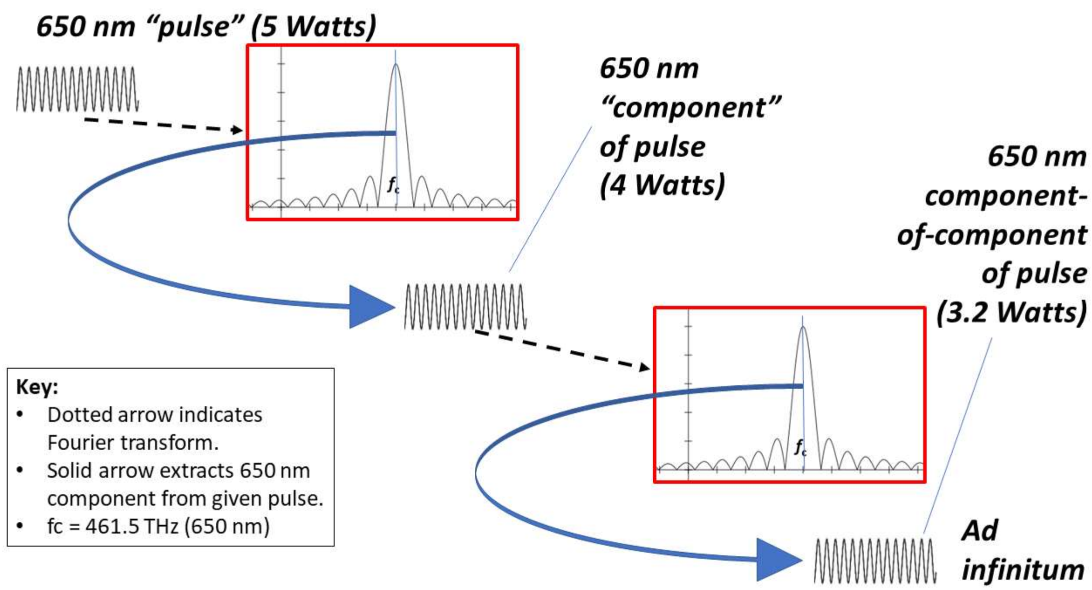

Figure 7.4: The Vicious Reflectance Regress

Figure 7.4 shows conceptually what happens to the "650 nm" content of any finite-duration pulse propagating toward or away from a surface. Because that content analytically regresses to zero average power,[19] pulse-reflectance diminishes to 0/0[20] at

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[19] That is to say, Figure 7.4 does not show 3 physical pulses, but rather my attempt to understand how much pulse content is dissipating or reflecting at 650 nm. That content is what regresses conceptually or analytically to zero.

[20] One might object that zero is never actually reached in an asymptotic regress; but if the ratio becomes one infinitesimally small value divided by another, then the reflectance value at every wavelength, on no matter what surface, becomes the perfectly efficient 1/1, which is absurd.

any given wavelength, and therefore possesses no meaning. Even should a scientist "measure" 4W at 650 nm in the reflected pulse, she is not measuring a "pulse," by the argument just given. Hilbertian pulse-reflectance is conceptually incoherent and therefore unworkable. The "Vicious Reflectance Regress" (VRR) described in this section is why Shinelanders refrain from uttering nominalist Reflectance Claims like, "Mirror A is a better reflector than mirror B"; for such claims are nonsensical. Shinelanders substitute Reflectance Claims with oblique locutions: "You will see mirror A better than B in the dark"; but to ask them why, in hopes of learning something *about the mirrors*, is a non-starter.[21]

IV.

In section III, I argued that mathematically nominalist claims about pulse-reflectance (the received view) are nonsensical. Pulse-reflectance is a contradiction in terms, like a non-cubical cube. Nominalist Reflectance Claims can be meaningfully uttered, however, if we redefine reflectance in a way that blocks the VRR. Such redefinition is the purpose of this section, after which I argue that the mathematics indispensable to my VRR-blocking definition of reflectance are non-conservative over physical theory (section V).

[21] I cannot address many pertinent objections to the reflectance regress in this space (see instead Danne 2020). I should answer the general objection, however, that I too severely demand a complete definition for reflectance. The mean kinetic energy (MKE) version of temperature, for example, is not completely definable over the vanishing sphere used to estimate molecular velocity (Smith 1998, 40-41), but (the objection goes) MKE temperature is a real property. I reply that the MKE problem is a good one to have: chasing asymptotic *accuracy*, or being unable to tack on more decimal places, instead of suffering asymptotic regress to an undefined value, as reflectance does.

Simply stated, I block the VRR by correcting a misguided assumption of the Shinelanders and of other adherents of Hilbertian reflectance. That assumption is that Equation 1 for average power applies to finite-duration signals, or pulses. While this assumption works in many branches of science, it is only a shorthand technique unsuitable for the *metaphysics* of property ascription,[22] as my arguments of section III suggest. The more orthodox, signal-theoretic definition of "average power" applies only to infinite-duration signals, since such signals (usually periodic) possess infinite energy computed as an integral over all time; finite-duration signals have zero average power under those same limits of integration (Haykin and Van Veen 1999, 20-21). The philosopher and Shinelander should heed this orthodox signal-theoretic rule, moreover, to avoid the VRR when ascribing reflectance.

Specifically, one can block the VRR by redefining reflectance from the per-wavelength efficiency of a surface to reflect pulses, to the per-wavelength efficiency of a surface to reflect the infinite-duration monochromes (Fourier harmonics) that superimpose into pulses. I have already employed this solution, moreover, in Figure 7.2. Each point on the solid or dotted trace of Figure 7.2 (c) represents a harmonic of that point's indicated wavelength and amplitude. All such phase-shifted harmonics (phase plot not shown) sum without remainder into the finite-duration "pulses" of Figure 7.2 (a) and (b). This harmonic redefinition of reflectance blocks the VRR, because harmonics never undergo "harmonic dispersion"; their frequency is always monochromatic, and so their bandwidth is always unity; it is the harmonic that would reflect "per wavelength" in

[22] Or of Humean constant-conjunction. Figure 7.4 implies that Humeans cannot define what optical phenomenon they perceive to be constantly conjoined at 650 nm.

the most literal, coherent, and non-regressive sense. When a scientist measures a "4W" reflection at 650 nm, in my account, she is measuring the portion of a 650 nm *harmonic* that is not cancelled-out by superposition with its neighboring harmonics. Thus there is no component regress to chase, as in Figure 7.4.

Granted, harmonic realism is controversial,[23] as would be the claim that real harmonics actually reflect from surfaces, but such controversy is beside the point. The point is that ascribing harmonic-reflectance blocks the VRR and restores our mathematically nominalist language about reflectance. Shinelanders who adopt my redefinition can converse in Reflectance Claims in good conscience, because ascribable and meaningful reflectance is harmonic-reflectance, not the pulse-reflectance of the received view. Although Fourier harmonics are mathematical, Reflectance Claims are not mathematical, because they reference a property, reflectance, which is not itself a mathematical object.

V.

Hence my thesis. Fourier analysis is non-conservative over physical theory, because conjoining Fourier analysis with the sentences N* of Shineland (about the average powers of pulses) yields mathematically nominalist Reflectance Claims N^0 and N^1 not derivable from N*: "This surface is reflective" (N^0), and "Surface A is more reflective than surface B" (N^1). I have made N^0 and N^1 plausible for Shinelanders by re-defining reflectance as harmonic-reflectance, thereby blocking the VRR that rendered N^0 and N^1 meaningless. Thus, while Shinelanders cannot expunge the Fourier transform

[23] See Wilson (2018, 233, 238), McGivern (2008, 68, n. 19), Liston (1994), and Sheldon (1985) for discussion.

from their reflectance theorizing[24] (on pain of launching the VRR and losing the reflectance concept entirely), they can at least now assert in good faith N^0 and N^1 as they develop stealth aircraft and manufacture laser mirrors. In the terminology of **Conservation** introduced in section I, the Fourier transform (MT*) is indispensable[25] to deriving nominalist claims N^0 and N^1 from MT* & N*, and so Fourier analysis violates **Conservation** and is non-conservative over physical theory N*.

Here Boyce (2020) might deny that Fourier analysis violates **Conservation**, because nominalist claim N^0 ("This surface is reflective") follows not from MT* & N*, but from M & MT* & N*, where M is a "mixed mathematical statement[]" (14) entailed by the conjunction of MT* and some non-mathematical statement.[26] Perhaps mixed-mathematical statement M is, "Light propagates isomorphically to transverse sinusoids,"[27] and from that mixed-mathematical assumption (along with MT* & N*, and perhaps additional mixed-mathematical assumptions), N^0 and N^1 follow, preserving the conservativeness of MT*.

[24] I am not claiming that Fourier analysis cannot be nominalized into Field's (1980) relations of **betweenness** and **congruence**. Liston (1993) doubts that such a nominalization could succeed, but my claim is different: we need harmonics to posit reflectance. I argue below in the main text that the property of harmonics by which they block the VRR is a mathematical property that Field's predicates do not nominalize away.

[25] Some may object that alternative mathematical formalisms besides Fourier analysis represent light propagation, undermining the claim that the Fourier transform is "indispensable" to reflectance ascription. I acknowledge this insight, but the onus is on the objector to show that such an alternative formalism blocks the VRR. For example: unlike the harmonic basis of Fourier analysis, wavelet bases are heterochromatic and thus launch the VRR instead of blocking it.

[26] Thanks to Kenneth Boyce for this insight (email communication).

[27] Important to the success of my argument is that this sentence M not be part of N*. I think this assumption is fair, however, because I see no reason why Shinelanders would *have* to assume M when manipulating Equation 1.

I deny, however, that the statement performing the regress-blocking work in my example (section IV) is “mixed-mathematical.” The M-statement of the previous paragraph I rename BL, because it is what Field (1980) calls a “‘bridge law[]’” (9), a sentence “that involve[s] the mathematical vocabulary and the physical vocabulary together” (10), and that renders a given portion of MT* (namely the transverse sinusoidal terminology) *applied* rather than pure. While arguing the point here would take me too far afield, one cannot allow bridge laws to trivially protect every sentence of MT* from proof of non-conservativeness, without begging the question about the conservativeness of applied mathematics (cf. Melia 2006, §1b).

Hence, I propose that the regress-blocking statement—derivable *entirely* from MT*—that facilitates derivations of N^0 and N^1 is:

> R: Pulses are superpositions of harmonics weighted and phase-shifted by the Fourier transform.

R is a tenet of Fourier analysis; that R applies to all physical transverse waves is no proof of its mixed-mathematicity, because bridge laws BL* bear that mixing burden. The BL’s should be considered *part* of applied MT*, just as Field (1980, 17) applies Zermelo-Fraenkel set theory (ZF) by axiomatizing urelements not originally part of ZF. Nor is R’s applicability to all physical transverse waves any indication of its non-mathematicity, for Shinelanders cannot assert R nominalistically. In discussing a given “harmonic” referenced by R, Shinelanders could refer to a signal whose consecutive periods are **congruent** with each other “forever and ever.” But in that case, talking about forever-and-ever just is mathematical language about the infinite.

I say "in that case," because Field (1980) otherwise welcomes nominalist language about the infinite.[28] He denies, for example, that asserting "'there are infinitely many grains of sand'" introduces into N* any abstracta like real numbers, sets of them, or functions on them (95). He likewise endorses nominalist assertions of the **continuity** of a scalar quantity (Chapter 8, §A), and explicitly defines an "Inf" predicate for infinite quantities (101-102).

But the harmonics in R differ from Field's (1980) infinitudes, in two ways. The first, I indicated in the previous paragraph: while there are no remarkable nominalistic implications of infinity in Field's examples, there are *significant* nominalistic implications (for reflectance ascription) that follow from the positing of infinite-duration harmonics. Namely, nominalistic *reference* to reflectance cannot be meaningful if the scientific theory of reflectance lacks infinite-duration harmonic language (sections III-IV). The second difference is that Field introduces infinitudes arbitrarily, as nominalist expressions for the ranges of predicates, or of available "space-time points . . ." (103). I introduce harmonics, on the other hand, entirely from MT* mathematics; harmonics are *mathematical* infinitudes because Shinelanders have no prior reason to suppose that pulse average powers (constituting the reflectance ratio) pertain most appropriately to infinite-duration propagations of light. There is no sentence within N* suggesting that light is infinitely durative. The motivation to mention "harmonics" arises entirely from MT*, to render nominalist claims N^0 and N^1 meaningful.

[28] By contrast, Jody Azzouni treats the infinite (Azzouni 1994, 3) and infinitesimal (Azzouni 2009, 161) as essentially mathematical. René Guénon (2001) holds yet a third view, that the "infinite" means all of existence, and so cannot be constrained to mathematics, or to parts of existence like time or space, and is anyway not a mathematical notion.

Perhaps the Fieldian can reply that regardless of how infinitudes are introduced into N*, they can be immediately nominalized; but I think that such a maneuver violates **Conservation** at an extra-logical level. If I learn *new* sentences in N* only by first *studying* MT* & N*, then it appears that the new sentences were not in fact derivable from N* alone. The Shinelanders had no conception of infinite light propagation before encountering my argument of this paper (they never studied orthodox signal theory, section IV) and for my interlocutor to stipulate *post hoc* that Shinelanders implicitly harbored such a conception all along is unfair. Their N* contained no Inf(x) predicate, and no musings about how much sand is on the beach. Shinelanders only study electromagnetic pulses.

One might counter-object that if Shinelanders decline to ascribe reflectance *because* they recognize the infinite regress of the VRR (section III), then infinitudes are part of N* after all. In reply, I can easily weaken the example to conclude that Shinelanders notice the *conceptual incoherence* of pulse-reflectance, without realizing that the regress of a pulse's "average power" goes on forever.[29] In one sense, I interpret my dialectic as turning one of Field's (1980) arguments against him. To downplay the Platonist intuition that mathematics describes the world because the world *is* mathematical, Field contends that mathematics works because it was invented "to apply to space and time . . ." (33-34). Exactly. Theorem R is mathematical, and not mixed-mathematical, because it has no precedent in N* at all; it was invented for the

[29] Nurida Boddenberg suggests (reading group meeting) that Shinelanders' recognition that the reflectance ratio could possess *any* value between 0 and 1 also suggests an implicit infinitude. In reply, I assume that the peculiarly dense Shinelanders simply do not think that way about ratios (*viz*. that they are continuous).

Shinelanders to make one of their property ascriptions coherent, not to describe any observation. Yes, contemporary Earthlings practice recreational (unapplied) mathematics, derive new theorems, and nominalize them without violating **Conservation**; but that is because Earthlings possess advanced nominalization techniques (e.g., for infinitudes) already. The Shinelanders labor in a more primitive theoretical framework than we.

VI.

In conclusion, I have tried to show that Fourier analysis is non-conservative over physical theory because the harmonic waveforms introduced to render surface reflectance conceptually coherent are infinitely durative in a *mathematical* sense of "infinite." Harmonic infinitude is mathematical for the Shinelanders, because the concept of infinitude is lacking from their N* theory. This assumption about Shinelander epistemology or psychology might be implausible, but if it is not impossible, then it is possible that some mathematics (like Fourier harmonics) are non-conservative over physical theory. Some readers might disapprove of my rendering conservativeness a contingent, epistemic, or psychological notion rather than a logical notion,[30] but I think that this conclusion follows from my Shinelander thought experiment.

[30] Melia (2006, §1b) argues that bridge laws are contingent, which is a short step from arguing that some applied mathematics are contingently conservative.

CHAPTER 8

TWO IMPLICATIONS OF COLOR OBJECTIVISM FOR THEOLOGICAL AESTHETICS*

1. Introduction

Is color a property of objects in the world, an electrochemical response in the brain, a combination of both, or none of the above? Could the answer to this question implicate theories of aesthetics and religious art? Whereas answering these questions adequately would require several books, I consider in this paper two theological-aesthetical implications of the first proposal—that color objectivism is true, and that human-visible colors are real properties of objects. The most influential contemporary account of color objectivism is David R. Hilbert's (1987; and Byrne and Hilbert 2003),[1] which reduces color ontologically to sets of dispositional[2] reflectances, reflectance being the per-wavelength efficiency of a surface to reflect light. I have recently argued, however, that Hilbert's definition of reflectance faces a particular metaphysical difficulty (Danne 2020), and it is through my resolution of this difficulty that I connect color objectivism to theological aesthetics. The difficulty, I argue, is that the received definition of

*Submitted to *Theology and Science* on February 7, 2021.

[1] Chirimuuta (2008, 563) credits Hilbert with being the "first" to propose the color ontology that I describe in the main text.

[2] Color need not be *dispositional* reflectance for my argument to go through; color could instead be the causal categorical base of reflectance (whatever that base is in the physical world), as Jackson (1996) argues. On the distinction between categorical and dispositional properties, see Schrenk (2017, Chap. 2).

reflectance is the per-wavelength efficiency of a surface to reflect finite-duration pulses[3] of light, but according to a well-documented empirical law of nature, pulses do not propagate "per-wavelength." Pulses propagate as envelopes of frequencies,[4] and the per-wavelength components of these envelopes, if finitely-durative, themselves propagate as envelopes, *ad infinitum*. Thus, because Hilbert defines reflective efficiency as the ratio of reflected to incident pulse average powers (per wavelength), but because the per-wavelength average power of any pulse conceptually regresses to zero (due to the "envelope regress" mentioned in the previous sentence; see section 2), the reflectance ratio at every wavelength asymptotically regresses to the undefined and meaningless value of 0/0, and I deny that any property with that value can be ascribed to a surface, much less serve as the ontological reduction base for a philosophical concept as basic and important (to some) as surface color.[5]

My solution to the reflectance regress is *prima facie* theological, because I argue that the nearest-to-hand solution for blocking the regress is to redefine reflectance from the disposition to reflect pulses of light, to the disposition to reflect the optical Fourier harmonics that superimpose without remainder into pulses of light. Fourier harmonics are infinite-duration monochromatic sinusoids, and so harmonic-reflectance is the per-wavelength disposition to reflect *only* eternally-propagating light; if color objectivism is

[3] By "pulse" I mean a finite-duration simple sinusoid or cosinusoid, as shown in Figure 8.2 of the appendix to this chapter. Advanced optics utilizes Gaussian and other shaped pulses, but these variations do not affect my argument.

[4] For an example of such an "envelope," see either the solid or dotted trace of Figure 8.1 below.

[5] Although I cannot here enumerate or summarize all of color objectivism's rivals (see the Introduction of Gert 2017 for an overview), I note that my thesis applies not just to Hilbert's account, but also to ontological reductions of color to the electromagnetic medium, or to a combination of medium-and-surface (as endorsed by Pasnau 2009).

true, then, and my redefinition of reflectance holds, then all surface colors *are* dispositions to reflect only eternally-propagating light, and this eternalist dimension of surface color has theological overtones. I argue that it reinforces the *Via negativa* theology of abstract art defended by Christopher Longhurst (2012), and that it renders dubious the allegedly divine attributes of the human artist identified by Paul Crowther (2016). Because neither of these implications follow from Hilbert's received view, and because color objectivism is a popular commonsense intuition (much of our language predicates colors of objects), and because (I shall argue) Hilbert's ontological reduction fails anyway, both adherents and critics of color objectivism have reason to consider my claims, and theorists of art and aesthetics—particularly theological aesthetics—would do well to reconsider color objectivist assumptions that they might have thought were neutral for their theories of aesthetics and religious art.

Color objectivism has garnered a number of critics before me, however, whose views I should summarize to distinguish my thesis from them. For my purpose is not to pile on criticism of color objectivism, but to examine the implications of a near-to-hand emendation of its ontological reduction. Indeed, I think that the problem I identify for Hilbert's ontological reduction supersedes the other leading criticisms, as I will now try to show.

Joshua Gert (2017), for example, revisits the longstanding *epistemic* controversy[6] that if there is a "unique green" (69)—"neither yellowish nor bluish" to any degree

[6] Longstanding since discussed by Hardin (1988, Chapter 2), Cohen, Hardin, and McLaughlin (2006), Tye (2006), Byrne and Hilbert (2007; 2004), and others.

(63)—and this color reduces to some reflectance value[7] at “503 nm” (69), we would nevertheless be unable to explain how we knew that unique green was located at 503 nm. The reason why is that color objectivists correlate reflectance values to the color experiences of normal observers in standard conditions,[8] but “that ten different completely normal human beings are likely to locate unique green at ten different places on the spectrum between 490 and 520 nm” (69). Thus, to claim that unique green reduces to reflectance at a specific wavelength, but that we can neither know which one, nor point out its instances, would be an embarrassment for the color objectivists. In response to this criticism, color objectivists sometimes analogize color to other properties like temperature, whose physicality we accept despite our epistemic shortcomings about them (Byrne and Hilbert 2004, 42-43), but not everyone accepts such analogies (cf. Tye 2006).

Frank Jackson (1996; 1998, Chap. 4) rejects Hilbertian color objectivism for a different reason, broached in footnote 2; he denies that color reduces to a surface disposition (reflectance), because color experience should be *caused* by surface colors,[9] and dispositions are non-causal. Just as fragility does not cause a vase to break when dropped, but rather the ceramic microstructure of the vase causes the shattering upon impact, so Jackson thinks that some yet-to-be empirically established surface property that is *not* Hilbert’s amalgamation of reflectances causes color experience and is color,

[7] Reflectance values are per-wavelength ratios of reflected to incident average optical power (see section 2), and so fall between 0 and 1 at each wavelength, 1 indicating 100% efficiency (e.g., all 5 watts of an incident beam reflect from the surface).
[8] See Hardin (1988, Chap. 2) for these parameters, outside the scope of this paper.
[9] For criticism, see McFarland and Miller (1998; 2000); I do not take up that debate in this paper.

ontologically speaking. Because the effect of categorical bases on color experience can only be *usefully* described along a per-wavelength dimension, however,[10] my argument supersedes this objection by raising it anew at the categorical base level.

My argument also supersedes Colin McGinn's (1996) objection, that colors cannot be dispositions because they do not *look* like dispositions, and that color dispositionalism therefore yields an absurd "error theory about color perception . . ." (537). Important to understanding this objection is that by the term "disposition," McGinn does not mean reflectance or any specific physical property *per se*, but rather the disposition *relating* such a "first-order physical property *P*"[11] to the human individual who perceives (e.g.) red (538). McGinn denies that objects look like they have the disposition "to produce color experiences" (537), for four main reasons: (i) we cannot see into the "nearby possible worlds" (541) implicitly referenced by the subjunctive conditionals in dispositional language about color; (ii) "color is perceived as intrinsic" rather than as relational to observers (542); (iii) experiences of red would themselves need to be seen (since dispositional redness in McGinn's account is irreducibly experiential), but they are not (542-543); and (iv) color dispositionalists remain "unable to say that an object looks red simpliciter," but only that it looks like the disposition to look like a disposition *ad infinitum* (543-544). *Contra* theses (i)-(iv), McGinn takes color to be "the very content of primitive visual experience" (540), and so from color dispositionalism he infers an unpalatable error theory of perception.

[10] A glance at any physical optics text (e.g., Born and Wolf 1999) reveals the enormity of the difficulty of discussing light propagation (outside of geometrical ray theory) without wavelengths.

[11] *P* being, for example, Hilbertian dispositional reflectance, or Jacksonian categorical microstructure (see previous paragraph).

Responding to McGinn (1996), reflectance physicalist (i.e., color objectivist) Alex Byrne (2001) differentiates a property's not appearing to be a disposition, from a property's appearing to not-be a disposition (242). He grants that "[i]t does not appear that the property green is the disposition to look green" (242), but he defends color dispositionalism (the perceptual kind, distinct from reflectance physicalism) by shifting emphasis away from the property and toward the object. Byrne remarks that "when one looks at a tomato, the tomato appears to have the disposition to look red, and it appears that the manifestation of this disposition is occurring"; but then he adds, "the fact that a tomato appears to have the disposition to look red does nothing at all to show that it appears that this disposition and the property red are identical" (244). Hence color error theory is a *non sequitur* in Byrne's (2001) account, because objects can look to have dispositional color without their color looking like a disposition, even if a particular kind of property (such as dispositional reflectance) strongly correlates to human color reports.

Said another way, Byrne (2001) attributes to McGinn (1996) the premise known as "*Revelation*" (Byrne 2001, 245), and Byrne (2001) denies that *Revelation* fairly undermines color dispositionalism (245). *Revelation* entails, in part, that "if the proposition that *p* concerns the nature of the colours and it is true, then it appears that *p*," but Byrne finds this premise indefensibly strong, since colors do not look like microstructure, or light waves, or even "yet to be formulated non-physical" properties (245). Thus *Revelation* suspends all talk about the nature of color properties (245), and so preempts reflectance physicalism in a question-begging manner. The better route to avoiding error theory, Byrne insists, is to recognize that "colours and the dispositions to look coloured typically go together" (244) in a correlative sense. Color objectivism is one such correlative theory (Byrne and Hilbert 2003).

Having summarized these three controversies about dispositional color, it can be seen that if the Hilbertian ontological reduction does not work,[12] then the epistemic concerns of Gert, the metaphysical concerns of Jackson, and the phenomenological concerns of McGinn are secondary, as important and insightful as they are. In the next section I detail my criticism of Hilbert's reduction, before considering counter-objections in section 3, and presenting my harmonic solution in section 4. Section 5 examines the two aforementioned theological-aesthetical implications of my solution, and section 6 concludes.

2. How the Received Definition of Reflectance Fails

The problem with defining reflectance as a dispositional property of surfaces originates in Hilbert's (1987) seminal formulation of it:

> There is a well-known dispositional property of objects . . . . This is the surface spectral reflectance of an object. . . . To measure the surface spectral reflectance . . . the ratio of the flux of incident light to the flux of reflected light is measured for each wavelength. Surface reflectances, thus conceived, are stable properties of objects. (Hilbert 1987, 1037-1041)[13]

[12] A fourth highly technical objection is that Maloney and Wandell (1986) have already argued that the SSR color reduction 'does not work' because the human visual system introduces errors into the apprehension and processing of SSR values in some conditions. Hilbert (1987, 2225 ff.) is well aware of this problem, but my SSR objection supersedes even Maloney and Wandell's, since they treat SSR as a surface property, a thesis which I deny!

[13] This wording may appear to contradict my remarks in footnote 7, since Hilbert in this block quote appears to locate incident average power (flux) in the numerator of the reflectance ratio; but this wording by Hilbert is a slight infelicity inconsistent with the rest of his book (Hilbert 1987). Parties to the color objectivism debate universally place reflected average power in the numerator of the reflectance ratio.

What I have hitherto called "reflectance," Hilbert calls "surface spectral reflectance" (SSR), the disposition of a surface to reflect fluxes of light, flux being the per-wavelength average power (in watts) of a light pulse (Hilbert 1987, 1033-1042). Human-visible colors reduce to sets of reflectances in the 400-800 nm range, in Hilbert's account, and color objectivists (Hilbert 1987, 267-277; Byrne and Hilbert 2004, 39-43) interpret this wavelength limitation as an anthropocentric characterization of SSR color rather than as a human dependency that renders color a subjective property. SSR obtains at surfaces, therefore, independently of their ambient illumination, and independently of the presence of humans.

The problem that I put to Hilbert's account resembles (but in many ways does not resemble) the problem of measuring a coastline: the smaller the unit, or the more resolution you employ to measure a coastline, the longer it gets (Smith 1998, Chap. 2). I generate a different but more severe metaphysical problem: by demanding ever more specificity about how much "flux" is reflecting at (e.g.) 650 nm, I discover that per-wavelength flux-reflectance (i.e., pulse-SSR, or Hilbertian SSR)[14] is a non-concept, because self-contradictory. Figure 8.1 below indicates why.

[14] For the rest of this paper, "pulse-SSR," "Hilbertian SSR," "flux-reflectance," and "Hilbertian reflectance" are all the same property.

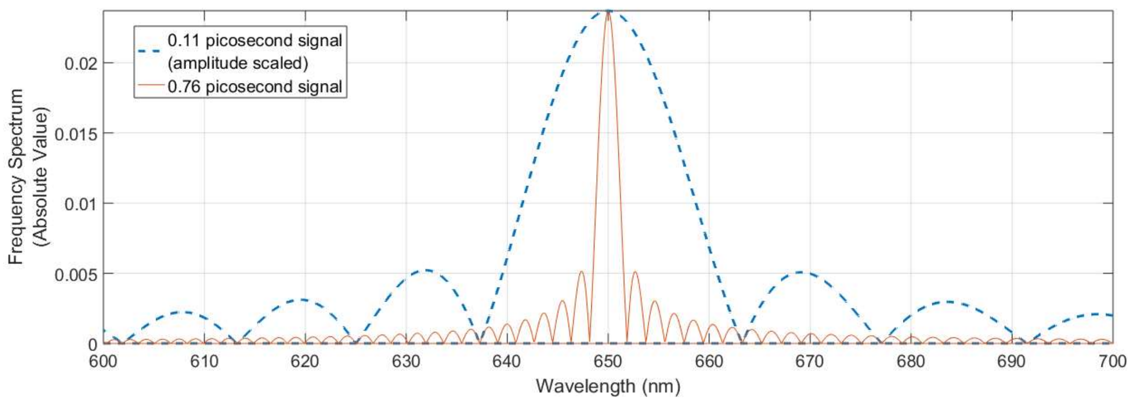

Figure 8.1: Harmonic Dispersion [7x duration]

Figure 8.1 illustrates the empirical law of classical electromagnetics that I mentioned in section 1. That law I call "harmonic dispersion," the inverse relationship of a pulse's duration to its bandwidth. Laser scientists using precision optical sources report pulse frequency[15] content similar to that shown in Figure 8.1 (Stingl et al. 1995; Deng et al. 2005), so harmonic dispersion should not be understood as any kind of "measurement noise," mathematical artifact, or quantum duality or uncertainty unfairly applied to non-quantum classical optics (Hirlimann 2005, 31).[16] The point of Figure 8.1 is that per-wavelength pulse-SSR is conceptually incoherent, because 0.11 picosecond (ps; 10^{-12} seconds) and 0.76 ps pulses of 650 nm monochromatic light dissipate considerably different amounts of "flux" at the wavelengths *surrounding* 650 nm. Whereas the longer pulse focuses most of its flux at 650 nm (solid trace), the shorter pulse exhibits

[15] Frequency and wavelength are interchangeable in physical optics; pulse oscillation frequency is the speed of light divided by wavelength.

[16] Harmonic dispersion transpires at all pulse durations and frequencies, including the macroscopic (millisecond) durations of radio and radar.

substantial flux peaks around 630 and 670 nm (dotted trace).[17] From this trend alone, the pulse-SSR value at any wavelength can be shown to regress asymptotically to 0/0, as I will now demonstrate.

Consider, for example, a 650 nm monochromatic source generating a 5 watt (W) pulse with duration 0.11 ps (as in Figure 8.1 dotted trace). Next imagine that this pulse propagates toward a perfect mirror, with value SSR=1 across the human-visible band. The key question is: Will this pulse reflect *all* 5W of its flux at 650 nm, on the perfect mirror? A glance at Figure 8.1 suggests not: some power will be lost to wavelengths besides 650 nm. For simplicity, assume that 20% is lost to side-frequencies, and 80%—or 4W—reflects at 650 nm (as will become clear, the accuracy of this estimate does not matter, because a regress is a regress). But now the even more critical question: will all 4W *really* reflect at 650 nm? The Hilbertian ostensibly needs to answer yes, because finite-duration "pulses" are what reflect per-wavelength; but I say no, because the 4W component isolated (dialectically, in this example) for reflection at 650 nm is itself only 0.11 ps long (what other duration would it have?). Thus, since 0.11 ps signals centered at 650 nm dissipate 20% of their flux to non-central wavelengths (Figure 8.1), then the 4W component actually dissipates only 3.2W at 650 nm, which by the same argument is actually 2.56 W (another 20% dispersed), which by the same argument is actually 2.05 W (another 20% dispersed), *ad infinitum*. Hence the SSR of the perfect mirror at 650 nm is not 1, but asymptotically approaches 0/0 (since the incident and reflected fluxes each regress), a meaningless value.

[17] The Fourier transform used to generate Figure 8.1 from sinusoidal pulses yields additional spectral content at negative frequencies, which I omit for clarity. See Haykin and Van Veen (1999, Chap. 3 and 4).

Important to note is that I have not shown this reflectance regress to be a *physical* regress, but I do not need to. A scientist measuring the "flux" at 650 nm likely measures 4W, and thinks nothing further of it, but my point is that she has not measured a "pulse," by the argument just given. Pulses do not reflect per-wavelength, so pulse-SSR is a meaningless property like "non-cubical cube." Color objectivists need something besides pulse-SSR to be the ontological reduction base of surface color; what could that something be?

3. Intermediate Objections, and Replies

Before considering what dispositional reflectance could be, I should set aside two of the most pressing objections to the reflectance regress.[18] The first is that I demand too much exactitude about the SSR concept. As the cartographer has to stop *somewhere* in his resolution scaling before measuring the coastline (section 2), so I have to stop chasing the per-wavelength flux dissipation. By a similar analogy, temperature understood as the mean kinetic energy (MKE) of substrate molecules also resists complete definition, since velocity is undefined through the vanishing imaginary sphere used to estimate molecular velocity (Smith 1998, 39-41). I reply that unlike the reflectance regress, the MKE regress is not vicious, because the MKE regress of the shrinking sphere *increases the accuracy* of a given temperature value by adding decimal places to the right. The reflectance regress, on the other hand, obliterates pulse-SSR entirely, sending its value to 0/0. In comparison to the reflectance regress, the MKE regress and cartographer's frustration appear rather benign.

[18] For additional objections and replies, see Danne (2020).

Secondly, it is no objection to claim that my argument of section 2 utilizes sub-picosecond light, a duration at which humans cannot discriminate colors (Scase and Foster 1988). For recall that I am not trying to critique *perceptual theory*, before I am trying to determine what reflectance could possibly be. Color objectivists insist that color reduced ontologically to reflectance is "intrinsic" to surfaces, or obtains independently of stimuli and observers (Byrne and Hilbert 2003, 9), but I argued in section 2 that every pulse-SSR value regresses to 0/0, and I cannot make sense of the claim that a property with that value is intrinsic to anything. With these important objections sidelined, I now proceed to analyze what reflectance could be.[19]

4. Redefining Dispositional Reflectance to be a Non-Regressive Property

As it turns out, a near-to-hand mathematical formulation stops the regress of section 2. The solution is to redefine reflectance from the per-wavelength disposition of a surface to reflect *pulses* of light (Hilbert's 1987 account), to the per-wavelength disposition of a surface to reflect the Fourier harmonics[20] that superimpose without remainder into pulses of light. Fourier harmonics are infinite-duration monochromatic sinusoids, and so the inverse relation of their duration to their bandwidth is always unity: harmonics are *immune* to harmonic dispersion, by definition. Hence a disposition to reflect harmonics would be the "stable" and non-regressive property that Hilbert (1987) seeks (section 2). I have already employed harmonics, moreover, in my construction of

[19] It pays also to point out that switching from a wave to a photonic model of light propagation does not affect my argument, since photonic absorbance and emission occur "per wavelength" in finite time, generating the same regress that afflicts waves.

[20] Or optical propagations with this structure. I remain intentionally ambiguous on the kind of harmonic (mathematical) realism in play, such as Aristotelian immanent universalism or Platonic transcendent universalism, since this specification does not matter for my first-pass analysis.

Figure 8.1. Every datapoint in the solid and dotted plots of Figure 1 represents the amplitude of a Fourier harmonic with the wavelength indicated for that point on the horizontal axis. All such harmonics represented by the solid or dotted trace of Figure 8.1 superimpose without remainder into the respective 0.11 ps or 0.76 ps sinusoidal pulses (not shown; see Haykin and Van Veen 1999, Chap. 3) that entered the analysis.[21]

According to my redefinition, then, when a scientist measures a 4W "flux" reflection at 650 nm, she is measuring some portion of a 650 nm *harmonic* not cancelled-out in superposition by its neighboring harmonics (see Figure 8.1). By (dis)analogy to the coastline paradox, I have found a measuring rod (the harmonic) that gives the *same* coastal length (reflectance value) regardless of measurement scale (pulse duration). More seriously, finite-duration pulses (Figure 8.2, chapter appendix) do not really exist as self-standing entities in my account. Phase-shifted[22] and weighted harmonics that propagate forever are what superimpose throughout the universe and give the appearance of finite-duration pulses. Granted, this solution is ontologically exotic, but I am not sure that it appears any stranger than calling "intrinsic" reflectance a property that possesses value

[21] A non-arbitrary and signal-theoretic reason for why harmonic-SSR does not regress is that computing the "flux" or "average power" of finite-duration signals as Hilbert (1987) does is a colloquial shorthand acceptable for much of applied science but not employed in pure signal theory. Orthodox signal theorists insist that only infinite-duration signals possess average power (Haykin and Van Veen 1999, 20-21), and incidentally this orthodoxy repairs Hilbert's (1987) ill-fated colloquialism in the realm of metaphysics. It is noteworthy, in this regard, that scientists do not rigorously ascribe "properties" like philosophers do, for there are no International Standard (SI) units for ascription, dispositionality, categoricity, intrinsicality, universals, tropes, instantiation, etc. Hence the philosopher must be on guard against defining real properties with applied science heuristics.

[22] The per-wavelength phase plot for the harmonics in Figure 8.1 is not shown, but it is needed for the harmonics plotted to superimpose into finite-duration pulses like those shown in Figure 8.2 of the chapter appendix.

0/0 at all wavelengths. At any rate, I see no immediate scientific objection to harmonic realism,[23] and so I assume harmonic-SSR as a first-pass solution for rendering color objectivism viable.[24] With that viability tentatively established, I can now examine the two theological-aesthetical implications of reducing color to harmonic-SSR, which I mentioned in section 1. For the theological tenor of the aesthetics question emerges prominently: *What would it mean for surface color to be the disposition to reflect infinitely-durative and only infinitely-durative light?*

5. Reflectance Physicalism and Theological Aesthetics

Without much effort, superficial connections could be forged between my harmonic-SSR color reduction and passing theological remarks about art, such as Felix-Jager's (2015, 86) quote of Nicholas Evans on the artist being "'in touch [. . .] with the infinite'" (brackets mine), or Karl Rahner's claim that

[23] Morganti (2013, 179) and Barrow (1998, Chap. 6) suggest that the universe could be spatially infinite, which in my view allows for the propagation of infinite-duration harmonics, in principle. Otherwise, harmonic realism is philosophically controversial: see Wilson (2018, 233, 238), McGivern (2008, 68, n. 19), Liston (1994), and Sheldon (1985). One might object *mathematically* that I unfairly gravitate to a Fourier representation of light, when other mathematical formalisms are available. I reply that the onus is on the objector to show that another formalism blocks the reflectance regress of section 2; wavelets, for example, which represent pulses more efficiently than harmonics do, are themselves heterochromatic, and so re-launch the reflectance regress instead of blocking it.

[24] Some might object that a disposition with eternal manifestation is no "disposition" at all. I follow Chakravartty (2013, 45) in postponing a rash judgment in this regard, but the details fall outside the scope of this paper. Others might contend that I am mistakenly treating as real what is an idealization. Ellis (1992, 276), for example, claims that monochromatic waves (harmonics) are an idealization necessary for *understanding* electromagnetic properties like reflectance. While idealization is a huge topic that I cannot fully address here, I reply that according to my regress argument of section 2, Ellis (1992) is simply mistaken about harmonics, because unless dispositional reflectance possesses infinite-duration harmonic manifestation, reflectance cannot be ascribed at all. More generally, I question how harmonics can be idealized without the human-visible colors of color objectivism being idealized.

> [r]eal artists undoubtedly announce what is eternal . . . . in a unique manner, in which their historical peculiarity and their longing for eternity are combined in a unity that constitutes the essence of the work of art. I may understand Dürer's hare as the most concrete aspect of a well-determined insignificant human experience, but when I look at it with the eyes of an artist, I am beholding, if I may say so the infinity and incomprehensibility of God. (Rahner 1992, 166)

The harmonic-SSR color objectivist (hereafter "harmonic color objectivist") can make truisms of both of these claims, by pointing out that *any* observer of color is "in touch with" or in visual reception of the harmonic infinitudes that are color's manifestation, and that any finite-duration viewing experience is eternally preordained by the superimposed harmonics that constitute the pulse impinging on the viewer's retina. I suspect that such connections multiply trivially for every mention of the infinite within theological aesthetics. Hence, I narrow my focus, and attempt to isolate theological-aesthetical theses to which harmonic color objectivism applies non-trivially, or systematically.

5.1 Supplementing Longhurst's Via Negativa through Abstract Monochromes

One promising application for harmonic color objectivism is the work of Christopher Longhurst (2012), who calls "theologically meaningful" (68) the abstract paintings *Black Square* by Kazimir Malevič, and *Abstract Painting no. 34* by Ad Reinhardt. The former painting consists of a sizeable black square on a white background, and the latter features a faintly discernible cross constructed of black squares against a black background. Longhurst finds these works theologically significant for their "apophatic dimension" (71), their utility for the "'*Via negativa*'" (70) that "bring[s] the observer closer to a sense of God through the path of abandonment, through absence,

[and] through what is not revealed" (72). These monochromes[25] facilitate the negative way, Longhurst argues, through their imageless form and particular color.

Specifically, Longhurst (2012) interprets the paintings' non-representational symbolic form as "an authoritative source for understanding Existence while colour, used as a subject, becomes a device to expound the attributes of God" (70). As imageless, the form of the paintings "restrain[s] the intellect from becoming distracted by what the eyes see" (72), and as Longhurst quotes Earle Coleman, the formal "'emptiness is not mere vacancy; by being devoid of finite things, empty space has "room" to receive and suggest an infinite presence'" (Longhurst 2012, 72, footnote omitted). The undistracted mind, through the form of the monochrome, encounters "the maximum possibility of being" (Longhurst 2012, 73), a maximum that Longhurst understands as theological.[26]

Hence even before considering how Longhurst (2012) takes color to "expound the attributes of God," I can anticipate that his project resists trivial applications of harmonic color objectivism like those mentioned in the first paragraph of section 5. Triviality is avoided in Longhurst's account because the form-*less*-ness of the monochromes expresses the infinite in a threefold way. Firstly, the paintings convey the infinite by their layout as paintings. Longhurst opines that "[t]he [monochrome] images appear voluminous and possessive of infinite space representing the divine attributes of infinity

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[25] "Monochrome" has two meanings in this paper: (i) a Fourier harmonic, (ii) a unicolor painting. I rely on context to disambiguate, where possible.

[26] Longhurst (2012) equates this maximum possibility of being with the transcendental attributes of "the beautiful, the Good and the True" (73), characteristics traditionally ascribed to God (see Viladesau 1999, Chap. 4). Such is not necessarily to say that Longhurst finds the monochromes beautiful, however, because as a *Via negativa*, the monochrome conveys instead that its object (God) is *not* ugly, *not* evil, lacking no power, and possessing no falsity (Longhurst 2012, 70-71).

and immensity," and that "the form reaches beyond the picture field . . ." (75). Such formal reaching beyond prompts Longhurst to claim that the paintings' spatial "scale is indeterminate . . ." (75).

Secondly, the monochromes express the infinite by prompting spiritual experiences in the viewer (see two paragraphs ago). Longhurst's insights about these experiences are numerous and complex, but he eventually claims:

> Such an aesthetic approach to theology maintains that one can never truly define the divinity in words or figures; that in the end reason gives way to visual sensation and intellectual discourse halts; that the rational functions of the human mind surrender to the contemplative mode of human being. Upon viewing such artwork, the passive observer becomes an active participant in something trans-temporal . . . .
> . . .
> [T]hese art-forms are capable of evoking a primordial awareness in the human mind providing access to the nonphysical, transcendent and spiritual aspects of reality. (Longhurst 2012, 75)

The transtemporal engagement of the viewer sounds like something that harmonic color objectivism facilitates, since color as the disposition to reflect harmonics is the disposition to reflect only eternally-propagating light (section 4). Presumably, we do not *experience* the infinitude of time when we look at a monochrome painting (how could we as finite beings?), but if harmonic color objectivism is true, then the monochrome painting maximizes the viewer's engagement in trans-temporality by maximizing the visible surface area of a *single* reflectance profile[27]—i.e., a single color—across the canvas. Looking at just one color involves us maximally in the transtemporal

[27] A "reflectance profile" is the set of reflectance values possessed by a surface, from 400-800 nm.

manifestation of a single reflectance profile, whereas a painting with multiple colors keeps us jumping (I allege) from one to the other.

Thirdly, monochromes express the infinite by virtue of those paintings emerging (in the opinion of Longhurst, Malevič, and Reinhardt) as "icons" of contemporary culture; as icons, Longhurst explains, monochromes "not only represent reality or direct to another reality, but they are, *in se*, their own reality" (Longhurst 2012, 70). Thus, the monochrome painting, as a formless but colored reality, *is* in an iconic and immediate sense the disposition to reflect eternal and only eternal light, if the color of the paint is ontologically harmonic-SSR.

But would a formless monochrome of *any* hue meet the three criteria of spatial breadth, facilitation of experience, and iconicity, to qualify as an apophatic vehicle for self-transcendence?[28] Longhurst suggests not, explaining that

> Black is . . . simultaneously the absence of all colours that make up light—a negative connotation, and the imparticipable colour, that is, a perfect combination of multiple colours—its positive meaning. It is in this latter description that black approaches closely the notion of the divine.
>
> Using the colour black along with a non-figurative pictorial medium Malevič and Reinhardt expose the absence of content with the infinity of space and the notions of immateriality and permanence. (Longhurst 2012, 77)

Thus, the specifically black monochrome intimates the divine by the black hue's imparticipability, which I understand as the inability of other hues to quite *be* or *contain* the black; just as Truth or Goodness alone fail to adequately communicate what God is,

[28] Barbara Rose (2006) suggests a positive answer when she quotes Yves Klein: "When you look at a terribly sad color, for instance, you feel swamped by it and dissolved in a space that is immeasurably sad and goes beyond all dimensions. It is a sad freedom vaster than infinity!" (Rose 2006, 190). Rose associates "blue" with "infinity" (9), and remarks that "the monochrome . . . opens onto an infinite perceptual depth . . ." (130).

and as predicating of God positive attributes yields less understanding than does denying of God the opposites of those attributes. The observer of a black monochrome apprehends its imparticipability in a negative way, Longhurst suggests, by experiencing the painting as lacking color, a lack which itself "leads to the meaning of an absence of possession, an emptiness and silence" that prompts theological reflection (Longhurst 2012, 77).

In sum, my contribution to Longhurst's (2012) account is that if black surface color *is* harmonic-SSR, then black monochromes impart to their viewers an additional dimension of the infinite that Longhurst overlooks; namely, the eternal dimension of *time* that makes harmonic-SSR the non-regressive disposition that it is. Of course, this infinite dimension obtains for monochrome paintings of *any* hue, but Longhurst provides special reason to appreciate the iconicity of black monochromes in the color objectivist framework. The point, however, is not merely that harmonic-SSR fortifies Longhurst's (2012) thesis while Hilbert's (1987) received view does not. Hilbert's received view of pulse-SSR color objectivism actually undermines Longhurst's thesis, because color *cannot* reduce to pulse-SSR (section 2), and so according to Hilbert's received view a monochrome painting cannot *possess* a black color, and so cannot *be* an icon, in my opinion.

5.2 Tempering the Creative Abilities of Crowther's Artist

The second synthesis of harmonic color objectivism and theological aesthetics that I perform in this paper involves the work of Paul Crowther (2016), who advances an ambitious thesis about man, God, the *Imago Dei*,[29] and pictorial art. Specifically, Crowther claims that "the very making of pictorial art involves metaphysical factors that can also be interpreted as disclosing aspects of humanity's relation to the Godhead" (143). Key to understanding these metaphysical factors is understanding Crowther's metaphysics of time and space, since one godly dimension that Crowther identifies in man is the ability to create spatiotemporal being itself through art.

Crowther (2016) understands spatial extension or "[s]pace occupancy," for starters, as the touchstone of being (144), and he discredits observer-*less* spatial extension as an "[in]determinate" (144-145) and "abstract" (144) concept. It is "the cognitive activity of an observer," Crowther explains, that determines space occupancy and renders it "character[izable]" or "meaningful" (145). Similarly, he considers observer-less tensed *time* to be unreal, attributing to "[s]elf-consciousness" the power to "introduce[] present, past, future, and possibility into the physical world" (144).

Notably, Crowther's (2016) observer imparts spatiotemporal reality to the universe by degrees. At a lower level of being reside imagined and remembered tensed states and possibilities of the universe (145), what may be called the "virtual" objects of imagination or memory (cf. 50, 145, 148). Committing virtual objects to the "physical medium" of a painting or other picture lends virtual objects a higher level of being (146). Of course, "an image of something does not cause its object to exist" (145), but according to Crowther every image

> double[s] space . . . through engaging with both the past and the realm of possibility. Through visual imagination and memory, space is projected in virtual terms, allowing the immediate spatial aspects of things to be made

[29] In not necessarily a Judeo-Christian sense. Crowther (2016, 50) assumes that "self-conscious and autonomous beings . . . exist in God's image . . . ."

> intelligible in relation to other spatial aspects—past, future, and possible. (Crowther 2016, 145)

Hence a mental image becomes spatiotemporal to some degree, not because it physically occupies a mass of neurons (if thoughts are physical),[30] but because its object tends to be imaged as it formerly extended or hypothetically would extend in space (145). This immersing of the virtually spatial in the past and possible is to "articulate[] space across the horizon of time" (146), and such an immersion into time always constitutes an uptick in the degree of reality of the object, an uptick in the "completion" of its existence (146).

Picturing (creating physical drawings) gives virtual objects a yet higher degree of reality than they enjoy in thought, therefore, not (merely) because the objects incur representation on a physical canvas, but because drawing the object "presents possibility at the same ontological level as the very criterion of something's existing—namely, spatial extension" (147). "[P]oems or stories" lack this ontological significance, Crowther contends,[31] whereas painting the virtual object invokes the temporal horizon needed to situate the spatial object in the "present . . ." (147). In Crowther's words, painting "concreti[zes]" the temporal horizon of the virtual object's past and possibility (148), and this concretization elevates the reality of the virtual object by two levels or degrees: one for becoming concretely spatial (on the canvas) and one for exhibiting the past or possible through those "spatial properties" (148). A painting of a historic log cabin, for example, depicts the cabin as present at some time, and as present in relation to

[30] By my reading, Crowder (2016) takes no position on the (im)materiality of thought (cf. 149).

[31] In part because "[t]exts . . . do not have to look like what they are talking about" (Crowther 2016, 146).

how future, past, or possible log cabins look spatially, or occupy space. By meta-immersing the virtual object in time and space, the painter renders the virtual object more real than it was in thought (148), and this human ability to "complete" the "being" (150) of a virtual object Crowther finds theologically significant, and even godly (150-160).

I demur that the harmonic color objectivist cannot likely accept the whole of Crowder's (2016) metaphysics of human agency (and vice versa), since the colors of the paint representing a given virtual object[32] cannot themselves stand in only a *finite* temporal horizon, or a horizon only as long as the artist's memory, the viewer's memory, the possible time conceivable by either, or the actual time of human history, etc. The temporal horizon in which the colored paint itself is situated must be infinite, lest the reflectance regress launch anew, and the color turn out not to be a property of the paint (sections 2 and 4).[33] Thus the human agent's creative self-consciousness is not as powerful as Crowther surmises, if colors really are surface reflectances. For surface color, if spatiotemporally real, cannot be the disposition to manifest for a duration of only remembered time, or finite possible time, or human-historical time, but only infinite time; and because Crowther appears to delimit time to the usable or practical horizon of an observer, an *infinite* temporal horizon and harmonic color objectivism appear precluded by his metaphysics.

Thus, when Crowther (2016) remarks that "the perception of [a painting's] virtual space involves an opening up of the temporal horizon through spatial properties" (148), he fails to say enough. The very supposition that colored paint constitutes the painting's

[32] I pass over the difficult question of whether imagined virtual objects should be said to possess surface colors.
[33] Crowther (2016) does not by my reading commit to any specific ontology of color.

virtual space opens a temporal horizon—an infinite one. Man's self-consciousness, as ostensibly finite, would not open this horizon; rather the existence of surface color opens this horizon. Man may manufacture colored paint and apply it to canvases, but that application in time is ostensibly not the conscious creation of infinite time. Real, infinite-duration time must instead transcend man's consciousness for objects to be colored, a conclusion at odds with Crowther's account.

6. Conclusion

In the first half of this paper, I argued that the received view of color objectivism (Hilbert 1987; Byrne and Hilbert 2003) is unsound, because pulse-SSR (the color objectivists' definition of reflectance) is conceptually incoherent, viciously regressive, and thus an implausible candidate for the ontological reduction base of surface color. The incoherence of pulse-SSR follows from its definitional opacity to harmonic dispersion, the inverse relationship of pulse duration to pulse bandwidth. I argued that redefining dispositional reflectance in terms of the Fourier harmonics that superimpose into pulses renders reflectance conceptually coherent and *pro tanto* suitable for supporting ontological reductions. This redefined reflectance property I call "harmonic-SSR," and the repaired version of color objectivism "harmonic color objectivism." To be sure, I have not shown harmonic-SSR to be a *unique* solution to the reflectance regress (section 2), but such a consideration is beside the point; the point is that the received view of reflectance needs repairing somehow, and that the expedient Fourier solution ramifies theological aesthetics.

In the second half of this paper, I considered two such ramifications. The first is that Longhurst's (2012) *Via negativa* to the divine through the observation of black monochromes garners additional support. I argued that harmonic color objectivism introduces a second infinitely-sized dimension to the paintings, the first being the infinite spatiality apprehended by Longhurst. I argued, in effect, that if the monochromes are icons (as their creators interpret them to be), and if their apophatic color *is* harmonic-SSR, then the monochromes are icons of the eternal, because they prominently exhibit the disposition to reflect only infinite-duration harmonics.

The second theological-aesthetic consequence of my thesis is more negative than the first. I argued that Crowther's (2016) metaphysics of space, time, and human artistry, which allows artists to create space and time through art, is incompatible with harmonic color objectivism. The reason why is because the surface color of an artist's paint, reduced ontologically to the disposition to reflect only infinite-duration harmonics, must temporally transcend the finite human artist who observes and manipulates the paint. While I do not know if Crowther is a color objectivist, some of his readers likely are, and so they should be made aware of that tension in his account, between personal creativity and color objectivism. I anticipate disclosing further theological applications of harmonic color objectivism in future work.

7. Appendix

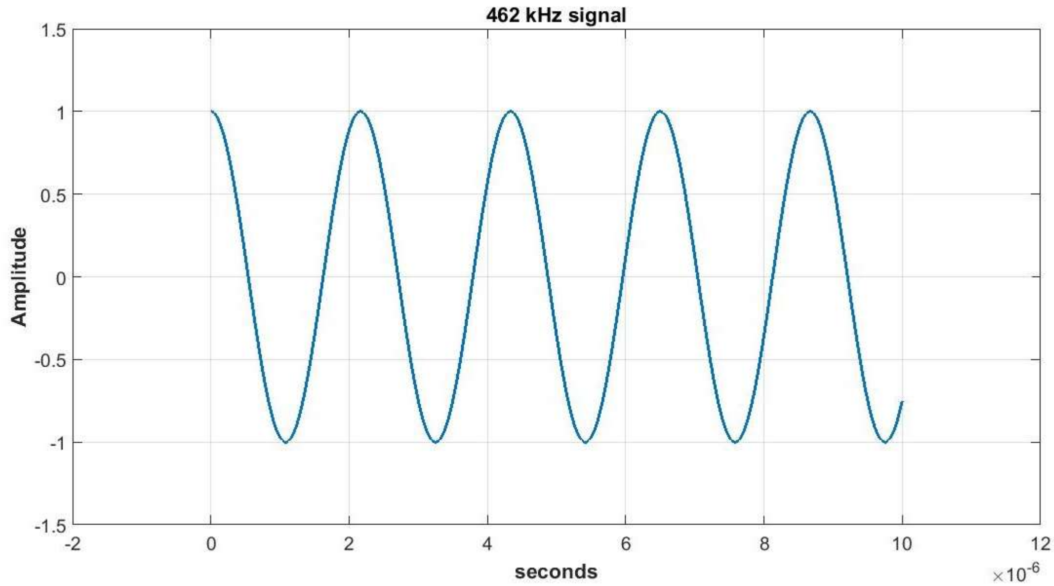

Figure 8.2: Simple Cosinusoidal Pulse

CHAPTER 9

CONCLUSION

As I interpret my foregoing contribution in the most general sense, I happened upon the puzzle of describing what are or constitute the per-wavelength "components" of any propagating electromagnetic wave packet. Whatever they are, I finally reasoned (in each chapter), they cannot just be clandestinely *nested* pulses of radiation,[1] since real pulses always *are* wave packets: the per-wavelength components of wave packets are wave packets, *ad infinitum*. Thus the per-wavelength magnitudes of pulses—intensity, energy, average power, instantaneous power—recursively and regressively redistribute to auxiliary wavelengths, until each magnitude is asymptotically zero, and the ratio of such magnitudes (representing reflectance) approaches the meaningless value 0/0. My solution to this puzzle, suggested in each chapter, is to take Fourier analysis at face value and interpret "per wavelength" magnitudes literally, as magnitudes of monochromatic harmonics of infinite duration. Depending on antecedent philosophical premises, this solution to the reflectance regress (chapter 2) reinforces structural realism (chapter 3), mathematical realism (chapter 4), and extra-mathematical program explanation (chapter 5). The solution frustrates, on the other hand, some extant accounts of fictional modeling in science (chapter 6), the strong thesis that "good" applied mathematics is conservative

[1] As a Russian doll has a smaller doll inside of it, so the 650 nm "component" of a finite-duration pulse seems to keep spawning sub-components at 650 nm (chapter 2).

(chapter 7), and yields mixed conclusions for theological aesthetics, if harmonic-reflectance is taken to be the ontological reduction base for surface color (chapter 8).

In retrospect, I acknowledge one counter-argument by which most if not all of the implications listed above (minus the reflectance regress itself) might be resisted.[2] That counter-argument is that harmonics do not really block the reflectance regress, because harmonics are not really "infinite." René Guénon (2001) argues that the *in-finite*, literally construed, is neither physical nor mathematical, but metaphysical, since the infinite would literally encompass all things, as nothing could limit it. Space, for example, is not infinite for Guénon, but is clearly limited by its own "determination" (14, n. 13), for what non-spatial thing would determine where space stopped? Analogously, he contends that the infinite is not a property of any number or set thereof; numbers rather go on indefinitely due to "the very nature of number in all its generality," a generality "formed from the finite" (13) and which is neither spatial nor temporal, etc.,[3] but numerical. Thus harmonics, whether interpreted as mathematical entities or as physical monochromes, possess only *indefinite* duration, in Guénon's account, and I am not sure whether indefinite harmonics block the reflectance regress; for the indefinite pulse would have to be longer than every finite-duration pulse (actually or in principle), and this thesis requires further research.

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[2] Except for the perennial counter-argument that harmonics are idealizations, and that I have not yet identified the *right* account of scientific idealization for my problem. I do not mean to disparage this objection, but I cannot now debate it further than I did in chapter 1.

[3] Infinite generality *is* spatial and temporal, on Guénon's view, so the numerical is not infinite.

In the meantime, if Guénon's interpretation of the infinite were strictly imposed, my arguments of chapters 3-5, and 7, which assume the infinite to be a mathematical concept, would collapse. Perhaps those arguments could be refurbished with the mathematical harmonic possessing *indefinite* duration, but my dialectical opponents Chakravartty, Azzouni, Saatsi, Field, and Boyce would need to be similarly convinced that the infinite is non-mathematical, for my arguments to be fair and relevant. The implications of Guénon's thesis for chapter 8 also require more consideration than I can provide here. Would abstract monochromes construed as "icons of the *indefinite*" still be portals to the divine? Can Crowther's agent bring "indefinite" manifestations of colored paint into temporal being? The distinction in play is subtle, and the aesthetical implications fall outside the purview of philosophy of science.

Possible next directions for my thesis follow on the conditionalization that I defended in chapters 4 and 5, between property realism and mathematical realism. An ambitious section from chapter 4 that died in journal peer review and was omitted from this dissertation included the claim that the intrinsicality versus extrinsicality of ascribed properties is stipulated by ascribers to obtain (and recall from chapter 4 how the deflationary nominalist loathes stipulation as the mark of the unreal!). My point was to criticize Azzouni's TEA criteria for surreptitiously allowing *all* property ascription to violate **Robustness**, since as ascribed and hence as stipulated to be intrinsic or extrinsic, properties never "diverge from how a theory predicts them to be, or from what observers 'believe about what they'll observe'" (cf. Azzouni 2004b, 383). While this claim was too strong to develop with the resources and space at hand, the conditionalization of mathematical realism on the *stipulative nature* of (some) property realism intrigues me.

Indeed, perhaps a feasible route to developing this thesis—that property realism and mathematical realism depend sometimes on the stipulative dimension of ascription—is to synthesize it with work by Scott Soames (2010). In a nutshell, Soames rejects traditional theories of meaning in which we perceive propositions and the meanings of sentences by a kind of platonic intuition, an intuition sensitive to propositions that intrinsically represent reality and thereby render our thoughts, speech, and sentences representational (7). Soames counters that there is nothing in any abstract structure itself—a structure that a proposition might instantiate—that unequivocally predicates any given property of any given subject or entity (31). Hence, he argues that "agents predicating properties" is "explanatorily prior" to the proposition, its meaning, and the meanings of sentences and speech, etc. (7). "[P]ropositions are representational *because* of the relations they bear to inherently representative mental states and cognitive acts of agents" (7). Likewise, I suspect that objects have properties *because* of ascribers' volitions. Whereas Azzouni finds his TEA criteria for posit existence compatible with property antirealism, I here press the different point that property *and* mathematical realism sometimes jointly rise and fall with ascribers' volitions (and that this result upsets TEA for a different reason than I argued in chapter 4). Until I commit and say that the mirror is "intrinsically" reflective, for example (and what scientist can agree or disagree with this claim?), belief in a literal harmonic manifestation of the mirror is unwarranted.[4]

My point is not to commit Azzouni to propositional analyses (which by my reading he largely avoids), or to charge Soames with solipsism about fundamental scientific properties like reflectance. For Soames (2010) is careful to argue that the truth

[4] This same argument applies for calling the mirror "extrinsically" reflective.

values of propositions[5] "are relativized to what is required to *entertain* them" (89), where "entertaining" a proposition is the action of predicating some "constituents" of a given propositional structure to other constituents (65). Because both true and false propositions are "*entertainable*" (90), predication in Soames's account is not infallible (114), and snow was white even when dinosaurs lived but no predicating human being existed (90). Building on these insights, I point out that conceptual regresses like the reflectance regress are anathema to the *entertainability* of propositions. Thus, sometimes the function of mathematical entities (harmonics) is not primarily to make a property ascription true or false, but to make that ascription entertainable at all. That a surface could even be reflective is what bestows ontological import on the constituent harmonics of that proposition, and that the surface *is* reflective—a volitional human stipulation—is what seals the deal in favor of harmonic realism. This contingent connection between some mathematics and entertainability appears to support the claim that some mathematics and properties come into being together, by the volitional power of the agent entertaining and ascribing them.

[5] Which differ from the truth values of sentences, for reasons outside the present discussion (Soames 2010, 89).

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APPENDIX A

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How to make reflectance a surface property

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Publication: Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics

Publisher: Elsevier

Date: May 2020

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